

Spin transfer torque in continuous textures: semiclassical Boltzmann approach

Frédéric Piéchon and André Thiaville

Laboratoire de Physique des Solides, Bât. 510, CNRS, Univ. Paris-sud, 91405 Orsay, France

We consider a microscopic model of itinerant electrons coupled via ferromagnetic exchange to a local magnetization whose direction vector $\mathbf{n}(\mathbf{r}, t)$ varies in space and time. We assume that to first order in the spatial gradients and time derivative of $\mathbf{n}(\mathbf{r}, t)$ the magnetization distribution function $\mathbf{f}(\mathbf{p}, \mathbf{r}, t)$ of itinerant electrons with momentum \mathbf{p} at position \mathbf{r} and time t has the Ansatz form $\mathbf{f}(\mathbf{p}, \mathbf{r}, t) = f_{\parallel}(\mathbf{p})\mathbf{n}(\mathbf{r}, t) + f_{1r}(\mathbf{p})\mathbf{n} \times \nabla_{\mathbf{r}}\mathbf{n} + f_{2r}(\mathbf{p})\nabla_{\mathbf{r}}\mathbf{n} + f_{1t}(\mathbf{p})\mathbf{n} \times \partial_t\mathbf{n} + f_{2t}(\mathbf{p})\partial_t\mathbf{n}$. Using the Landau-Sillin equations of motion approach we derive explicit forms for the components $f_{\parallel}(\mathbf{p})$, $f_{1r}(\mathbf{p})$, $f_{2r}(\mathbf{p})$, $f_{1t}(\mathbf{p})$ and $f_{2t}(\mathbf{p})$ in "equilibrium" and in out-of-equilibrium situations for (i) no scattering by impurities, (ii) spin-conserving scattering and (iii) spin non-conserving scattering. The back action on the localized electron magnetization from the out-of-equilibrium part of the two components f_{1r} , f_{2r} constitutes the two spin transfer torque terms.

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I. INTRODUCTION

Recent experiments in spin-valve nanopillars^{1,2}, point contacts³ and ferromagnetic nanowires^{4,5,6,7,8} have demonstrated the possibility to "manipulate" the magnetization by applying an electrical current instead of an external magnetic field. It is believed that this phenomenon might give rise to many technological applications, (MRAM, fast magnetic switching, high quality microwave sources), provided that the current density necessary to manipulate the magnetization can be drastically reduced.

This possibility of electrical current induced magnetization manipulation was predicted already ten years ago by J. C. Slonczewski and L. Berger^{9,10}.

Nowadays, the consensus is that, in presence of an electrical current, the standard Landau-Lifshitz-Gilbert (LLG) equation that describes the dynamics of the magnetization is modified. For ferromagnetic wires in which the magnetization texture $\mathbf{M}(\mathbf{r}, t) = M_s\mathbf{n}(\mathbf{r}, t)$ has a time and space dependent direction $\mathbf{n}(\mathbf{r}, t)$, the modified LLG equation reads¹¹:

$$\partial_t\mathbf{n} = \gamma\mathbf{B}_{\text{eff}}(\mathbf{n}) \times \mathbf{n} + \alpha\mathbf{n} \times \partial_t\mathbf{n} - \mathbf{u} \cdot \nabla_{\mathbf{r}}\mathbf{n} + \beta\mathbf{n} \times \mathbf{u} \cdot \nabla_{\mathbf{r}}\mathbf{n}. \quad (1)$$

where $\mathbf{B}_{\text{eff}}(\mathbf{n}) = \frac{-1}{M_s} \frac{\delta\mathcal{E}(\mathbf{n})}{\delta\mathbf{n}}$ is the effective magnetic field derived from the magnetic free energy and u , β are two phenomenological parameters where \mathbf{u} is proportional to the electrical current density and polarization, and α is the usual phenomenological Gilbert damping parameter. The contribution $\mathbf{u} \cdot \nabla_{\mathbf{r}}\mathbf{n}$ is called the adiabatic term and can be derived from an additional term in the magnetic free energy that takes into account the coupling to the electrical current¹². The β term is in contrast non adiabatic and appears to play a role similar to the Gilbert damping term (and one conception of the spin-transfer torque introduction in the LLG equation even predicts $\beta = \alpha$ exactly¹³). Recent micromagnetic numerical experiments^{14,15,16,17} with this modified LLG equa-

tion have greatly clarified the qualitative roles played by the two phenomenological parameters u and β . In particular, it has been proved that in the absence of the β term there is no current induced steady domain wall motion below a finite (very high) critical current density u_c . For a non zero β term, in the simple case of a perfect sample it can be shown that the speed of the domain wall is $(\beta/\alpha)u$. More quantitatively, results compatible with the various experiments are obtained for a ratio $\beta/\alpha \geq 1$ equivalent to $\beta \simeq 10^{-2}$.

The derivation of the LLG equation above usually rests on a two steps argument. It is assumed that an itinerant ferromagnet can be modeled as a two "components" system: (i) non moving and ferromagnetically ordered electrons (called hereafter d) described by a classical magnetization vector $\mathbf{n}(\mathbf{r}, t)$ that varies slowly with time t and position \mathbf{r} ; (ii) current-carrying itinerant electrons (called s) coupled to the d electrons via a ferromagnetic exchange energy $\Delta_{sd} > 0$ such that the effective one-electron quantum Hamiltonian has the form $\hat{H}(\mathbf{r}) = -\frac{\hbar^2}{2m}\nabla_{\mathbf{r}}^2\hat{I} + \frac{\Delta_{sd}}{2}\mathbf{n}(\mathbf{r}, t) \cdot \hat{\boldsymbol{\sigma}}$, where \hat{I} is the 2×2 identity matrix and $\hat{\boldsymbol{\sigma}}$ is the vector of Pauli matrices with eigenvalues ± 1 .

The first step consists in finding the quantum average itinerant electron magnetization $\mathbf{m}(\mathbf{r}, t) = -\mu_B\text{Tr}\{\hat{\rho}(\mathbf{n})\hat{\boldsymbol{\sigma}}\}$ as a function of the "quasistatic" $\mathbf{n}(\mathbf{r}, t)$ ($\hat{\rho}(\mathbf{n})$ is the itinerant electron density matrix that depends on $\mathbf{n}(\mathbf{r}, t)$). In a second step one substitutes the resulting $\mathbf{m}(\mathbf{r}, t)$ back into the LLG equation of the localized electrons magnetization:

$$\partial_t\mathbf{n} = \gamma\mathbf{B}_{\text{eff}}(\mathbf{n}) \times \mathbf{n} + \alpha\mathbf{n} \times \partial_t\mathbf{n} + \frac{\gamma}{M_s\mu_B}\Delta_{sd}\mathbf{m}(\mathbf{r}, t) \times \mathbf{n}. \quad (2)$$

For domain walls, Zhang and Li (ZL)¹¹ were the first to make a transparent derivation along this line of reasoning. They found that in out-of-equilibrium situation the above back action not only produces the two spin transfer torque terms but also leads to correction of the Gilbert damping term and gyromagnetic ratio. Prior to

ZL, Zhang, Levy and Fert¹⁸ had obtained corresponding results for spin-valves. Both works however rest on a phenomenological equation of motion for the magnetization $\mathbf{m}(\mathbf{r}, t)$ of itinerant electrons where key ingredients are put by hand, especially spin flip scattering time and adiabaticity of the itinerant electron spin current with respect to local magnetization $\mathbf{n}(\mathbf{r}, t)$. Beside these phenomenological descriptions, various microscopic approaches have also flourished in the last years. For spin-valves systems several groups used a scattering matrix approach (for a review see¹⁹). For ferromagnetic wires the concept of local spin reference frame was often invoked so as to exhibit a direct coupling between spin current of itinerant electrons and gradient of the local magnetization $\mathbf{n}(\mathbf{r}, t)$ ^{12,13,20,21,22,23}. To the best of our understanding however, none of these different works really succeeds to establish the modified LLG equation as written above.

Very recently two independent works, by Tserkovnyak *et al.*²⁴ and Kohno *et al.*²⁵, based on different theoretical techniques presented a direct microscopic derivation of the two additional spin torque terms. Both works show that the β term requires the existence of a spin flip like scattering mechanism (e.g. spin non-conserving scattering like spin orbit, magnetic impurities ...). They show that this mechanism is also responsible for the appearance of an effective α Gilbert damping term induced by the itinerant electrons. In fact Tserkovnyak *et al.* argue further that one should find $\alpha = \beta$ for itinerant ferromagnetic systems where the magnetism comes from the exchange interaction between the itinerant electrons (e.g. the effective local $\mathbf{n}(\mathbf{r}, t)$ is in fact the itinerant electron magnetization $\mathbf{m}(\mathbf{r}, t)$ itself). The results of Kohno *et al.* were obtained from diagrammatic linear response theory and concerned only "integrated" physical quantities like the local magnetization $\mathbf{m}(\mathbf{r}, t)$. In contrast Tserkovnyak *et al.* used the Keldysh quasiclassical Green function technique that in principle could allow determining the full magnetization distribution function $\mathbf{f}(\mathbf{p}, \mathbf{r}, t)$, and thus might give a deeper understanding of the system.

In this paper we reconsider the entire problem within a Boltzmann approach. This provides an intuitive and hopefully pedagogical semiclassical picture of the equations of motion of the charge and spin distribution functions of the itinerant electrons, in a space-time dependent magnetization field $\mathbf{n}(\mathbf{r}, t)$. Our main assumption is that around any time space position the direction of $\mathbf{n}(\mathbf{r}, t)$ can be arbitrary but its gradients $\nabla_{\mathbf{r}}\mathbf{n}$ and $\partial_t\mathbf{n}$ must be slow enough so that only terms parametrically linear in these gradients are important (e.g terms like $\nabla_{\mathbf{r}}\mathbf{n}$, $\partial_t\mathbf{n}$, $\mathbf{n} \times \nabla_{\mathbf{r}}\mathbf{n}$ and $\mathbf{n} \times \partial_t\mathbf{n}$). This parametrization is different from that of Kohno *et al* and Tserkovnyak *et al.* which is in fact a linear theory around the uniform magnetization case (e.g. $\mathbf{n}(\mathbf{r}, t) = \hat{z} + \mathbf{u}(\mathbf{r}, t)$ with $|\mathbf{u}(\mathbf{r}, t)| \ll 1$). The difference might appear subtle, but it leads to distinct properties already in the equilibrium situation as compared to Tserkovnyak *et al.*.

The Boltzmann method provides a complementary physical picture that is intermediate between the purely

phenomenological macroscopic equations of motion approach of ZL on one side and the less intuitive but more microscopic quantum linear response or Keldysh methods on the other side. More precisely, on the one hand, since we deal directly with the charge and spin distribution functions, we have a deeper understanding than that provided by ZL method. We have also access to more microscopic physical quantities, and can in principle study general AC and thermoelectric effects not accessible to ZL method³⁵. On the other hand, the physical picture provided by the Boltzmann method is more transparent than the microscopic approach, it also allows understanding more clearly at which level spin-flip scattering differs qualitatively from spin-conserving scattering. Lastly, the Boltzmann method provides the natural framework to see where quantum corrections could be important. Indeed, in a forthcoming paper we will reexamine our results using the Keldysh Green function method in the quasiclassical approximation. This constitutes the natural theoretical framework to build from first principles the equations of motion for the distribution function in the presence of elastic and inelastic collisions. The Keldysh approach appears necessary because the construction of the collision integral in the Boltzmann picture is purely phenomenological so that, on a more microscopic level, it is not clear if there are important quantum and gradient corrections to the Boltzmann collision integrals.

The paper is organized as follows. In the first section, going back to textbooks^{26,27} we rederive the collisionless transport equations for the charge and magnetization distribution to first order in time-space gradients. We show that without any collision the magnetization distribution function $\mathbf{f}(\mathbf{p}, \mathbf{r}, t)$ is not collinear to $\mathbf{n}(\mathbf{r}, t)$ and that, in particular, there is a contribution collinear to $\mathbf{n} \times \nabla_{\mathbf{r}}\mathbf{n}$ that gives rise to a finite equilibrium spin current. In the next two sections we study the influence of elastic scattering by impurities. Assuming Boltzmann type collision integrals, we first consider the effect of spin-conserving collisions, in equilibrium and in out-of-equilibrium situation. Already at equilibrium we obtain surprising results for the components of the distribution that are not collinear to $\mathbf{n}(\mathbf{r}, t)$. We show in particular that the equilibrium spin current is rotated in the direction $\nabla_{\mathbf{r}}\mathbf{n}$ by an angle θ that depends on the ratio between the elastic scattering time and the effective Larmor time. In out-of-equilibrium situation we find correspondingly a component of the magnetic distribution function collinear to $\nabla_{\mathbf{r}}\mathbf{n}$ that could in principle give rise to a β term as back action after \mathbf{p} momentum integration. (Un)fortunately it vanishes after \mathbf{p} momentum integration as spin conservation imposes. We then consider collisions that lead to spin flip. For the case of a uniform magnet, starting from the known form of the collision integral of each eigen-spin distribution, we build a collision integral invariant under spin basis change. Extending phenomenologically this collision integral to the case of a non uniform ferromagnet we describe how the equilibrium and out-of-equilibrium properties are modified by the spin flip scattering. In

particular, from the different components of the itinerant electron magnetization we evaluate their back action on the d electron magnetization and then extract explicit expressions for the induced Gilbert damping α_{2t} , modified adiabatic torque term u and finally the β torque term. Our expressions for α_{2t} and β coincide exactly with ZL approach and to leading order also coincide with results of Kohno *et al* for spin-isotropic spin flip scattering, when appropriate changes of notations are made. Concerning the parameter u , that is purely phenomenological in the ZL approach, our result is similar to that of Kohno *et al*. Note finally that, in order to improve reading, most of the calculations concerning spin flip scattering are put in the Appendix whereas the main text essentially provides the physical picture emerging from the modified distribution functions. In a last concluding section we discuss possible extensions of our approach to itinerant ferromagnets or ferromagnetic Fermi liquids^{28,29} and spin-valve systems³⁰.

II. SEMICLASSICAL TRANSPORT THEORY

A. Model and Ansatz

The effective one-electron quantum Hamiltonian of the itinerant free electrons coupled to the localized electron magnetization is of the form

$$\hat{H}(\mathbf{r}) = [-\frac{\hbar^2}{2m}\nabla_{\mathbf{r}}^2 + V(\mathbf{r})]\hat{I} + \frac{\Delta_{sd}}{2}\mathbf{n}(\mathbf{r},t) \cdot \hat{\boldsymbol{\sigma}}, \quad (3)$$

where $V(\mathbf{r}) = -e\mathbf{E}\mathbf{r}$ is the potential induced by an external uniform electric field. The intrinsic difficulty to find the equilibrium and out-of-equilibrium density matrix $\rho(\mathbf{r},\mathbf{r}',t)$ associated to this Hamiltonian originates from the non commutation of the "Zeeman" term with the kinetic term, due to the spatial variation of $\mathbf{n}(\mathbf{r},t)$.

However, as we consider here domain walls where the characteristic length of the magnetization gradient is large (10-100 nm) compared to the electron mean free path, quantum transport is not pertinent and electron diffusion is a more appropriate framework. Therefore we simplify the problem by assuming that the spatial degrees of freedom \mathbf{r}, \mathbf{p} are classical commuting variables and not quantum operators, and retain only the non trivial commutation rules of spin degrees of freedom. The effective semiclassical Hamiltonian of the itinerant electrons is thus

$$\hat{H}(\mathbf{r}, \mathbf{p}) = [\frac{\mathbf{p}^2}{2m} + V(\mathbf{r})]\hat{I} + \frac{\hbar\omega_{sd}}{2}\mathbf{n}(\mathbf{r},t) \cdot \hat{\boldsymbol{\sigma}}, \quad (4)$$

where we have denoted by $\omega_{sd} = \frac{\Delta_{sd}}{\hbar}$ the effective Larmor frequency. We further define $\tau_{sd} = \frac{1}{\omega_{sd}}$, and $\ell_{sd} = v_F\tau_{sd}$ the Larmor time and Larmor length respectively (v_F is the Fermi velocity).

The semiclassical quantity that corresponds to the density matrix is now the spinor distribution function

$$\hat{f}(\mathbf{p}, \mathbf{r}, t) = \frac{1}{2}[f(\mathbf{p}, \mathbf{r}, t)\hat{I} + \mathbf{f}(\mathbf{p}, \mathbf{r}, t) \cdot \hat{\boldsymbol{\sigma}}]. \quad (5)$$

The physical quantities such as local particle density $n(\mathbf{r},t)$, particle current density $\mathbf{j}(\mathbf{r},t)$, magnetization density $\mathbf{m}(\mathbf{r},t)$ and spin-current tensor density $\mathbf{J}(\mathbf{r},t)$ are obtained by integration on these distributions:

$$\begin{aligned} n(\mathbf{r},t) &= \int d\tau \text{Tr}_{\sigma} \{ \hat{f}(\mathbf{p}, \mathbf{r}, t) \} \\ &= \int d\tau f(\mathbf{p}, \mathbf{r}, t), \\ \mathbf{j}(\mathbf{r},t) &= \int d\tau \frac{\mathbf{p}}{m} f(\mathbf{p}, \mathbf{r}, t) \\ \mathbf{m}(\mathbf{r},t) &= -\mu_B \int d\tau \text{Tr}_{\sigma} \{ \hat{f}(\mathbf{p}, \mathbf{r}, t) \hat{\boldsymbol{\sigma}} \} \\ &= -\mu_B \int d\tau \mathbf{f}(\mathbf{p}, \mathbf{r}, t) \\ \mathbf{J}(\mathbf{r},t) &= -\mu_B \int d\tau \frac{\mathbf{p}}{m} \mathbf{f}(\mathbf{p}, \mathbf{r}, t) \end{aligned} \quad (6)$$

where

$$\int d\tau \equiv \int \frac{d\mathbf{p}}{(2\pi\hbar)^3} \equiv \int d\epsilon_{\mathbf{p}} \nu(\epsilon_{\mathbf{p}}) \int \frac{d\hat{\mathbf{p}}}{4\pi}$$

with $\hat{\mathbf{p}}$ the unit vector of direction \mathbf{p} , $\epsilon_{\mathbf{p}} = \frac{\mathbf{p}^2}{2m}$ and $\nu(\epsilon) = \frac{\sqrt{2m^3\epsilon}}{2\pi^2\hbar^3}$ the 3D free electrons density of states.

Our main assumption is that around any space-time position \mathbf{r}, t , the direction $\mathbf{n}(\mathbf{r},t)$ can be arbitrary but its gradients $\nabla_{\mathbf{r}}\mathbf{n}$ (resp. $\partial_t\mathbf{n}$) must be slow enough compared to the Larmor length ℓ_{sd} (resp. Larmor time) so that only terms linear in these gradients are important. Linearization in these gradients implies that time and space dependencies of the matrix distribution function $\hat{f}(\mathbf{p}, \mathbf{r}, t)$ are expanded on the possible directions $\mathbf{n}(\mathbf{r},t)$, $\mathbf{n} \times \nabla_{\mathbf{r}}\mathbf{n}$, $\mathbf{n} \times \partial_t\mathbf{n}$, $\nabla_{\mathbf{r}}\mathbf{n}$, and $\partial_t\mathbf{n}$. The Ansatz form we assume for $\hat{f}(\mathbf{p}, \mathbf{r}, t)$ compatible with this approximation is :

$$\begin{aligned} f(\mathbf{p}, \mathbf{r}, t) &= f(\mathbf{p}), \\ \mathbf{f}(\mathbf{p}, \mathbf{r}, t) &= f_{\parallel}(\mathbf{p})\mathbf{n}(\mathbf{r},t) \\ &\quad + \ell_{sd} [f_{1r}(\mathbf{p})\mathbf{n} \times \nabla_{\mathbf{r}}\mathbf{n} + f_{2r}(\mathbf{p})\nabla_{\mathbf{r}}\mathbf{n}] \\ &\quad + \tau_{sd} [f_{1t}(\mathbf{p})\mathbf{n} \times \partial_t\mathbf{n} + f_{2t}(\mathbf{p})\partial_t\mathbf{n}]. \end{aligned} \quad (7)$$

Note that $(\mathbf{n}, \partial_t\mathbf{n}, \mathbf{n} \times \partial_t\mathbf{n})$ and $(\mathbf{n}, \nabla_{\mathbf{r}}\mathbf{n}, \mathbf{n} \times \nabla_{\mathbf{r}}\mathbf{n})$ constitute two distinct orthogonal bases for any spin vector. Thus *a priori* our Ansatz contains some redundancy since $\partial_t\mathbf{n}$, $\mathbf{n} \times \partial_t\mathbf{n}$ are linear combinations of $\nabla_{\mathbf{r}}\mathbf{n}$, $\mathbf{n} \times \nabla_{\mathbf{r}}\mathbf{n}$ and reciprocally. The main reason why our Ansatz is nevertheless appropriate is that, in this extended basis $(\mathbf{n}, \partial_t\mathbf{n}, \mathbf{n} \times \partial_t\mathbf{n}, \nabla_{\mathbf{r}}\mathbf{n}, \mathbf{n} \times \nabla_{\mathbf{r}}\mathbf{n})$, to leading order, each component of the spin distribution is stationary (for DC field), space independent, and depends only on \mathbf{p} ³³. Had we chosen the basis $(\mathbf{n}, \partial_t\mathbf{n}, \mathbf{n} \times \partial_t\mathbf{n})$ to expand the spin distribution, each component would still be position-dependent to leading order.

Note that with our normalization the components f_{1r} , f_{2r} , f_{1t} and f_{2t} do have the same physical dimension as f_{\parallel} . As a consequence, the quantity $m_{1r}(\mathbf{r},t) = -\mu_B \int d\tau f_{1r}(\mathbf{p}, \mathbf{r}, t)$ has the units of a magnetization density.

Following Landau-Sillin²⁶, to first order in the gradients of the distribution function, the "Liouville" equation of motion of the distribution is obtained from

$$\frac{d\hat{f}}{dt} \equiv \partial_t \hat{f} + \frac{1}{i\hbar} [\hat{f}, \hat{H}]_- + \frac{1}{2} \{\hat{f}, \hat{H}\} - \frac{1}{2} \{\hat{H}, \hat{f}\} = \hat{\mathcal{I}}[\hat{f}], \quad (8)$$

where $[A, B]_{\mp}$ denotes the commutator (resp. anticommutator) of A and B and $\{A, B\} = \nabla_{\mathbf{r}} A \nabla_{\mathbf{p}} B - \nabla_{\mathbf{p}} A \nabla_{\mathbf{r}} B$ designates the classical Poisson brackets. $\hat{\mathcal{I}}[\hat{f}]$ represents the spin operator for the collision term. In the absence of collision, we obtain the following coupled set of equations of motion for the particle and spin components of the distribution function:

$$\begin{aligned} (\partial_t + \frac{\mathbf{p}}{m} \nabla_{\mathbf{r}} + e \mathbf{E} \nabla_{\mathbf{p}}) \mathbf{f} - \frac{\hbar \omega_{sd}}{2} \nabla_{\mathbf{r}} \mathbf{n} \cdot \nabla_{\mathbf{p}} \mathbf{f} &= 0, \\ (\partial_t + \frac{\mathbf{p}}{m} \nabla_{\mathbf{r}} + e \mathbf{E} \nabla_{\mathbf{p}}) \mathbf{f} - \frac{\hbar \omega_{sd}}{2} \nabla_{\mathbf{r}} \mathbf{n} \nabla_{\mathbf{p}} \mathbf{f} &= 0, \\ -\omega_{sd} \mathbf{n} \times \mathbf{f} &= 0. \end{aligned} \quad (9)$$

In these equations the symbols \times and \cdot mean vector product and scalar product of spin-vectors. For spatial quantities written in bold an implicit scalar product is understood, namely $\frac{\mathbf{p}}{m} \nabla_{\mathbf{r}} \equiv \sum_i \frac{p_i}{m} \nabla_{r_i}$ and $\nabla_{\mathbf{r}} \mathbf{n} \cdot \nabla_{\mathbf{p}} \mathbf{f} \equiv \sum_i \nabla_{r_i} \mathbf{n} \cdot \nabla_{p_i} \mathbf{f}$. For later use, we further write $\hat{f} = \hat{f}^0 + \hat{g}$ to separate the "equilibrium" ($E = 0$, but \mathbf{n} may depend on t) contribution \hat{f}^0 from the out-of-equilibrium contribution \hat{g} ; and accordingly for each component.

B. equilibrium properties

In the case of no applied electric field, the above equations have simple solutions.

a. Homogenous ferromagnet: $\mathbf{n}(\mathbf{r}, t) \equiv \mathbf{n}_0$ In the absence of an electric field and for a time and space independent direction $\mathbf{n}(\mathbf{r}, t) = \mathbf{n}_0$ the stationary distributions are well known:

$$\begin{aligned} f^0(\mathbf{p}) &= \frac{1}{2} (n_F(\epsilon_{\mathbf{p}}^+) + n_F(\epsilon_{\mathbf{p}}^-)), \\ \mathbf{f}(\mathbf{p}) &= f_{\parallel}^0(\mathbf{p}) \mathbf{n}_0, \\ \text{with} \\ f_{\parallel}^0(\mathbf{p}) &= \frac{1}{2} (n_F(\epsilon_{\mathbf{p}}^+) - n_F(\epsilon_{\mathbf{p}}^-)), \end{aligned} \quad (10)$$

where $\epsilon_{\mathbf{p}}^{\pm} = \frac{\mathbf{p}^2}{2m} \pm \frac{\hbar \omega_{sd}}{2}$ and $n_F(\epsilon) = (e^{\beta(\epsilon - \mu)} + 1)^{-1}$ is the Fermi statistic. The physical quantities are then:

$$\begin{aligned} n(\mathbf{r}, t) &= n_e = \int d\epsilon \nu(\epsilon) f^0(\epsilon), \\ \mathbf{m}(\mathbf{r}, t) &= m_{\parallel} \mathbf{n}_0, \\ \text{with} \\ m_{\parallel} &= \mu_B P n_e = -\mu_B \int d\epsilon \nu(\epsilon) f_{\parallel}^0(\epsilon), \end{aligned} \quad (11)$$

that defines the polarization P . Note that the scalar functions f^0 and f_{\parallel}^0 depend in fact of the energy ϵ .

b. Inhomogenous ferromagnet $\mathbf{n}(\mathbf{r}, t)$ The naive extension to a space and time dependent $\mathbf{n}(\mathbf{r}, t)$ would be an unchanged particle distribution and a magnetization distribution that follows $\mathbf{n}(\mathbf{r}, t)$ adiabatically: $\mathbf{f}(\mathbf{p}, \mathbf{r}, t) = f_{\parallel}^0(\mathbf{p}) \mathbf{n}(\mathbf{r}, t)$. (Un)fortunately it is not the solution of

the equations of motion above. Instead, to linear order in the space-time gradients, the solution is of the form:

$$\begin{aligned} \mathbf{f}(\mathbf{p}, \mathbf{r}, t) &= f_{\parallel}^0(\mathbf{p}) \mathbf{n}(\mathbf{r}, t) \\ &+ \ell_{sd} f_{1\mathbf{r}}^0(\mathbf{p}) \mathbf{n} \times \nabla_{\mathbf{r}} \mathbf{n} + \tau_{sd} f_{1t}^0(\mathbf{p}) \mathbf{n} \times \partial_t \mathbf{n} \end{aligned} \quad (12)$$

with

$$\begin{aligned} f_{1t}^0(\mathbf{p}) &= -f_{\parallel}^0(\mathbf{p}), \\ f_{1\mathbf{r}}^0(\mathbf{p}) &= -\frac{\tau_{sd}}{\ell_{sd}} \frac{\mathbf{p}}{m} f_{\perp}(\epsilon), \end{aligned} \quad (13)$$

and

$$f_{\perp}(\epsilon) = f_{\parallel}^0(\epsilon) - \frac{\hbar \omega_{sd}}{2} \partial_{\epsilon} f^0(\epsilon). \quad (14)$$

The above defined function $f_{\perp}(\epsilon)$ will appear many times in this paper and constitutes a key ingredient of most transverses magnetic quantities. The figure below plots this function $f_{\perp}(\epsilon)$ compared to $f_{\parallel}^0(\epsilon)$.

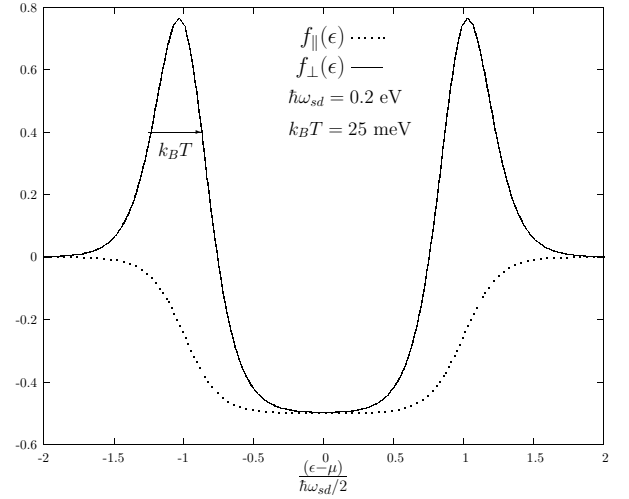


FIG. 1: The two functions $f_{\perp}(\epsilon)$ and $f_{\parallel}^0(\epsilon)$ for parameters $k_B T = 0.025 \text{ meV}$ and $\hbar \omega_{sd} = 0.2 \text{ eV}$.

The physical consequences of these non adiabatic components are the following. The itinerant electrons magnetization is $\mathbf{m}(\mathbf{r}, t) = m_{\parallel} \mathbf{n}(\mathbf{r}, t) + \tau_{sd} m_{1t} \mathbf{n}(\mathbf{r}, t) \times \partial_t \mathbf{n}$ with $m_{1t} = -m_{\parallel}$. As already pointed out by Zhang and Li¹¹, in the $s-d$ picture, the back action from the component m_{1t} renormalizes the γ term of the d electrons magnetization LLG equation. The component $f_{1\mathbf{r}}^0(\mathbf{p})$ does not contribute to the magnetization because of its rotational symmetry in momentum space. However it contributes to a local equilibrium spin current tensor $\mathbf{J}(\mathbf{r}, t) = \ell_{sd} \mathbf{J}_{1\mathbf{r}} \mathbf{n}(\mathbf{r}, t) \times \nabla_{\mathbf{r}} \mathbf{n}$ with

$$\begin{aligned} J_{1j}^i &= -\mu_B \int d\tau \frac{\tau_{sd}}{m} f_{1j}^0(\mathbf{p}) \\ &= \delta^{ij} \frac{2\tau_{sd}}{3m\ell_{sd}} \mu_B \int d\epsilon \nu(\epsilon) \epsilon f_{\perp}(\epsilon) \end{aligned} \quad (15)$$

where we used the identity $\int \frac{d\mathbf{p}}{4\pi} \hat{\mathbf{p}}^i \hat{\mathbf{p}}^j = \frac{\delta^{ij}}{3}$. In the $s-d$ picture, this equilibrium spin current has no back action.

In the itinerant ferromagnet picture where conduction electrons are responsible for the local magnetization itself (e.g. Δ_{sd} is the exchange interaction between the electrons), the divergence of this equilibrium spin current should be equivalent to the local exchange field term, as pointed out by several authors^{31,32}. This equilibrium spin current cannot be derived from the ZL macroscopic approach. As we will show in a forthcoming paper, it is also obtained with the quasiclassical Keldysh approach.

III. SPIN-CONSERVING IMPURITY SCATTERING

A. Collision integral and transport equations

We now consider the scattering by impurities in a purely phenomenological manner. For spin-conserving scattering, a first guess is a local collision integral of the form

$$\begin{aligned} \hat{\mathcal{I}}[\hat{f}(\mathbf{p}, \mathbf{r}, t)] &= \mathcal{I}\hat{I} + \mathcal{I} \cdot \hat{\sigma} \\ &= \int d\tau' w_{\mathbf{p}, \mathbf{p}'} \hat{f}(\mathbf{p}', \mathbf{r}, t) (1 - \hat{f}(\mathbf{p}, \mathbf{r}, t)) \\ &\quad - w_{\mathbf{p}, \mathbf{p}'} \hat{f}(\mathbf{p}, \mathbf{r}, t) (1 - \hat{f}(\mathbf{p}', \mathbf{r}, t)), \end{aligned} \quad (16)$$

where $w_{\mathbf{p}, \mathbf{p}'}$ is the probability to scatter from momentum \mathbf{p} to momentum \mathbf{p}' . In the expression above $(1 - \hat{f}(\mathbf{p}, \mathbf{r}, t))$ means $(1 - f)\hat{I} - \mathbf{f} \cdot \hat{\sigma}$. In general $\mathbf{f}(\mathbf{p}', \mathbf{r}, t)$ can be non collinear to $\mathbf{f}(\mathbf{p}, \mathbf{r}, t)$. As a consequence, due to the Pauli matrices properties, the above form leads to an unphysical imaginary contribution like $i(w_{\mathbf{p}, \mathbf{p}'} + w_{\mathbf{p}, \mathbf{p}'})(\mathbf{f}(\mathbf{p}', \mathbf{r}, t) \times \mathbf{f}(\mathbf{p}, \mathbf{r}, t)) \cdot \hat{\sigma}$. To prevent such a contribution it is necessary to antisymmetrize correctly the previous expression:

$$\begin{aligned} \hat{\mathcal{I}} &= \frac{1}{2} \int d\tau' w_{\mathbf{p}, \mathbf{p}'} \left[\hat{f}(\mathbf{p}', \mathbf{r}, t), (1 - \hat{f}(\mathbf{p}, \mathbf{r}, t)) \right]_+ \\ &\quad - w_{\mathbf{p}, \mathbf{p}'} \left[\hat{f}(\mathbf{p}, \mathbf{r}, t), (1 - \hat{f}(\mathbf{p}', \mathbf{r}, t)) \right]_+. \end{aligned} \quad (17)$$

With this form, the equations of motion for the charge and spin components of the distribution function become:

$$\begin{aligned} (\partial_t + \frac{\mathbf{p}}{m} \nabla_{\mathbf{r}} + e\mathbf{E} \nabla_{\mathbf{p}}) f - \frac{\hbar \omega_{sd}}{2} \nabla_{\mathbf{r}} \mathbf{n} \cdot \nabla_{\mathbf{p}} \mathbf{f} &= \mathcal{I}, \\ (\partial_t + \frac{\mathbf{p}}{m} \nabla_{\mathbf{r}} + e\mathbf{E} \nabla_{\mathbf{p}}) \mathbf{f} - \frac{\hbar \omega_{sd}}{2} \nabla_{\mathbf{r}} \mathbf{n} \cdot \nabla_{\mathbf{p}} \mathbf{f} \\ - \omega_{sd} \mathbf{n} \times \mathbf{f} &= \mathcal{I}, \end{aligned} \quad (18)$$

with $\mathcal{I} = \int d\tau' w_{\mathbf{p}, \mathbf{p}'} (f(\mathbf{p}') - f(\mathbf{p}))$ (similarly for \mathcal{I}) and where we use the property that for elastic scattering $w_{\mathbf{p}, \mathbf{p}'} = w_{\mathbf{p}', \mathbf{p}}$.

Whatever the form of $w_{\mathbf{p}, \mathbf{p}'}$, by integrating these equations of motion over \mathbf{p} the resulting continuity equations for the particle density $n(\mathbf{r}, t)$ and local magnetization $\mathbf{m}(\mathbf{r}, t)$ are the same as in the collisionless situation in absence of electric field, namely:

$$\begin{aligned} \partial_t n(\mathbf{r}, t) + \text{div} \mathbf{j}(\mathbf{r}, t) &= 0, \\ \partial_t \mathbf{m}(\mathbf{r}, t) + \text{div} \mathbf{J}(\mathbf{r}, t) - \omega_{sd} \mathbf{n}(\mathbf{r}, t) \times \mathbf{m}(\mathbf{r}, t) &= 0. \end{aligned} \quad (19)$$

As a consequence, the magnetization in the presence of spin-conserving scattering and DC electric field driving the system out-of-equilibrium cannot have finite components for either $m_{2\mathbf{r}} \nabla_{\mathbf{r}} \mathbf{n}$ or $m_{2t} \partial_t \mathbf{n}$. Thus, in that situation, as back action there are no induced α and β torque terms. We stress that this somewhat disappointing result concerning the nonexistence of a β term (finite $m_{2\mathbf{r}}$) for macroscopic quantities is not characteristic of what happens to the microscopic distribution function. In fact, as we explain below, at the level of the spin distribution distribution function, in the presence of spin-conserving impurities in out-of-equilibrium situation, there is a finite energy resolved magnetization $m_{2\mathbf{r}}(\epsilon)$ that results after angular momentum integration (or a finite angular momentum resolved magnetic density tensor $m_{2\mathbf{r}}(\hat{\mathbf{p}})$ that results after energy integration). So, the itinerant electron magnetization is not far from having a finite $m_{2\mathbf{r}}$.

Spin-conserving scattering by impurities produces other interesting results that we now describe in both equilibrium and out-of-equilibrium situations. With our general Ansatz linear in the gradients, we obtain the following set of coupled equations for each "stationary" component:

$$\begin{aligned} e\mathbf{E} \nabla_{\mathbf{p}} f &= \mathcal{I}, \\ e\mathbf{E} \nabla_{\mathbf{p}} f_{\parallel} &= \mathcal{I}_{\parallel}, \\ e\mathbf{E} \nabla_{\mathbf{p}} f_{2\mathbf{r}} + \frac{1}{\ell_{sd}} \left(\frac{\mathbf{p}}{m} f_{\parallel} + \frac{\hbar \omega_{sd}}{2} \nabla_{\mathbf{p}} f \right) + \omega_{sd} f_{1\mathbf{r}} &= \mathcal{I}_{2\mathbf{r}}, \\ e\mathbf{E} \nabla_{\mathbf{p}} f_{1\mathbf{r}} - \omega_{sd} f_{2\mathbf{r}} &= \mathcal{I}_{1\mathbf{r}}, \\ e\mathbf{E} \nabla_{\mathbf{p}} f_{2t} + \frac{1}{\tau_{sd}} f_{\parallel} + \omega_{sd} f_{1t} &= \mathcal{I}_{2t}, \\ e\mathbf{E} \nabla_{\mathbf{p}} f_{1t} - \omega_{sd} f_{2t} &= \mathcal{I}_{1t}, \end{aligned} \quad (20)$$

The components f, f_{\parallel} are completely decoupled from the others. By contrast, $f_{1\mathbf{r}}, f_{2\mathbf{r}}$ are coupled and depend on f, f_{\parallel} , and f_{1t}, f_{2t} are coupled and depend only on f_{\parallel} .

B. equilibrium properties

To go further we need to specify the form of the probability $w_{\mathbf{p}, \mathbf{p}'}$. To simplify the calculation we only consider the case of isotropic scattering $w_{\mathbf{p}, \mathbf{p}'} = w(\epsilon_{\mathbf{p}}) \delta(\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{p}'})$. As usual we define the inverse scattering time τ by:

$$\frac{1}{\tau(\epsilon_{\mathbf{p}})} \equiv \int \frac{d\mathbf{p}'}{(2\pi\hbar)^3} w(\epsilon_{\mathbf{p}}) \delta(\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{p}'}) = w(\epsilon_{\mathbf{p}}) \nu(\epsilon_{\mathbf{p}}) \quad (21)$$

and $\eta(\epsilon_{\mathbf{p}}) = \frac{\tau_{sd}}{\tau}$. The ratio η is anticipated to be much smaller than unity. Nevertheless, the calculations will be performed for a general η . With this specific form and in the absence of an electric field the previous expressions of $f(\mathbf{p}), f_{\parallel}(\mathbf{p}), f_{1t}(\mathbf{p})$, and $f_{2t}(\mathbf{p}) = 0$ are still solutions; their corresponding collision integral vanishes because all these distributions are function of $\epsilon_{\mathbf{p}}$ only. By contrast, the previous form of $f_{1\mathbf{r}}(\mathbf{p})$ is such that for a general $w_{\mathbf{p}, \mathbf{p}'}$ (and in particular for isotropic scattering), the corresponding collision integral $\mathcal{I}_{1\mathbf{r}}$ does not vanish. It thus implies a non vanishing $f_{2\mathbf{r}}(\mathbf{p})$ component that may modify $f_{1\mathbf{r}}(\mathbf{p})$ as a back action. One finds that the

new self consistent equilibrium solutions $f_{1\mathbf{r}}^0(\mathbf{p})$, $f_{2\mathbf{r}}^0(\mathbf{p})$ are:

$$\begin{aligned} f_{1\mathbf{r}}^0(\mathbf{p}) &= -\frac{\tau_{sd}}{\ell_{sd}} \frac{\mathbf{p}}{m} f_{\perp}(\epsilon), \\ f_{2\mathbf{r}}^0(\mathbf{p}) &= -\eta \frac{\tau_{sd}}{\ell_{sd}} \frac{\mathbf{p}}{m} f_{\perp}(\epsilon), \end{aligned} \quad (22)$$

where f_{\perp} is as given in (14) but divided by $1 + \eta^2$. The main modification induced by the spin-conserving scattering is thus that, in equilibrium, the spin current has now two components: a component collinear to $-\mathbf{n}(\mathbf{r}, t) \times \nabla_{\mathbf{r}} \mathbf{n}$ and a component collinear to $-\nabla_{\mathbf{r}} \mathbf{n}$, of smaller amplitude by a factor $\eta = \frac{\tau_{sd}}{\tau}$. The total modulus of the equilibrium spin current is smaller by a factor $\frac{1}{\sqrt{1+\eta^2}}$ compared to the collisionless situation. Apart from this change of modulus, the scattering has thus induced a rotation in spin space of the equilibrium spin current vector. The ratio between the two transverse components of the current defines a rotation angle θ by $\tan \theta = \eta = \frac{\tau_{sd}}{\tau}$. Let us stress that it is rather unusual to find any modification of the equilibrium distribution by elastic scattering; it might be a hint that some "quantum" corrections have been neglected.

From now on, to simplify the expressions, we assume that $\tau(\epsilon)$ is constant and independent of energy. When necessary, we comment on the validity of our results for an energy dependent $\tau(\epsilon)$.

C. out-of-equilibrium properties

Let us now consider the effect of the electric field. In the presence of an infinitesimal electric field, to each previous component $f_{\alpha}(\mathbf{p})$ will be added an out-of-equilibrium component $g_{\alpha}(\mathbf{p}) \equiv g_{\alpha}(\hat{\mathbf{p}}, \epsilon)$. To linear order in electric field and for isotropic scattering, standard calculations lead to the solutions

$$\begin{aligned} g(\mathbf{p}) &= -e \mathbf{E} \frac{\mathbf{p}}{m} \tau \partial_{\epsilon} f^0(\epsilon), \\ g_{\parallel}(\mathbf{p}) &= -e \mathbf{E} \frac{\mathbf{p}}{m} \tau \partial_{\epsilon} f_{\parallel}^0(\epsilon). \end{aligned} \quad (23)$$

The components g_{1t}, g_{2t} are solutions of the following coupled equations:

$$\begin{aligned} \frac{1}{\tau_{sd}} g_{\parallel} + \omega_{sd} g_{1t} &= \frac{1}{\tau} \int \frac{d\hat{\mathbf{p}}'}{4\pi} (g_{2t}(\hat{\mathbf{p}}', \epsilon) - g_{2t}(\hat{\mathbf{p}}, \epsilon)), \\ e \mathbf{E} \nabla_{\mathbf{p}} f_{1t}^0 - \omega_{sd} g_{2t} &= \frac{1}{\tau} \int \frac{d\hat{\mathbf{p}}'}{4\pi} (g_{1t}(\hat{\mathbf{p}}', \epsilon) - g_{1t}(\hat{\mathbf{p}}, \epsilon)). \end{aligned} \quad (24)$$

The solutions are immediate:

$$\begin{aligned} g_{1t}(\mathbf{p}) &= -g_{\parallel}(\mathbf{p}), \\ g_{2t}(\mathbf{p}) &= 0. \end{aligned} \quad (25)$$

The last two components are solutions of the following coupled equations:

$$\begin{aligned} e \mathbf{E} \nabla_{\mathbf{p}} f_{2\mathbf{r}}^0 + \frac{1}{\ell_{sd}} \left(\frac{\mathbf{p}}{m} g_{\parallel} - \frac{\hbar \omega_{sd}}{2} \nabla_{\mathbf{p}} g \right) + \omega_{sd} g_{1\mathbf{r}} &= \frac{1}{\tau} \int \frac{d\hat{\mathbf{p}}'}{4\pi} (g_{2\mathbf{r}}(\hat{\mathbf{p}}', \epsilon) - g_{2\mathbf{r}}(\hat{\mathbf{p}}, \epsilon)), \\ e \mathbf{E} \nabla_{\mathbf{p}} f_{1\mathbf{r}}^0 - \omega_{sd} g_{2\mathbf{r}} &= \frac{1}{\tau} \int \frac{d\hat{\mathbf{p}}'}{4\pi} (g_{1\mathbf{r}}(\hat{\mathbf{p}}', \epsilon) - g_{1\mathbf{r}}(\hat{\mathbf{p}}, \epsilon)). \end{aligned} \quad (26)$$

The general solutions of these equations are linear in E and have the parametric form:

$$\begin{aligned} g_{1\mathbf{r}} \mathbf{n} \times \nabla_{\mathbf{r}} \mathbf{n} &\equiv g_{1i}^j \hat{E}^j \mathbf{n} \times \nabla_{r_i} \mathbf{n} \\ \text{with} & \\ g_{1i}^j &= \frac{\tau_{sd}^2}{\ell_{sd}} \frac{e|E|}{m} [\hat{\mathbf{p}}^i \hat{\mathbf{p}}^j x_1(\epsilon) + \delta^{ij} y_1(\epsilon)] \end{aligned} \quad (27)$$

and similarly for $g_{2\mathbf{r}} \nabla_{\mathbf{r}} \mathbf{n}$. For the quantities $x_{1,2}(\epsilon)$ and $y_{1,2}(\epsilon)$ we obtain finally:

$$\begin{aligned} x_1 &= \frac{1+3\eta^2}{\eta} \frac{2\epsilon}{1+\eta^2} \partial_{\epsilon} f_{\perp} \\ x_2 &= 2\eta^2 \frac{2\epsilon}{1+\eta^2} \partial_{\epsilon} f_{\perp} \\ y_1 &= \frac{\eta}{3} x_2 + \frac{1+2\eta^2}{\eta} f_{\perp} - \frac{1}{\eta} f_{\parallel}^0 \\ y_2 &= -\frac{\eta}{3} x_1 - f_{\perp} \end{aligned} \quad (28)$$

Even if $g_{1\mathbf{r}}$ and $g_{2\mathbf{r}}$ are not simple distributions, from their rotational symmetry properties in momentum space we deduce that there is no finite spin current associated. By contrast, it is not clear if any finite magnetization will survive after angular and energy integration. Thus, it appears interesting to define two partial magnetization densities for each components. On the one hand, the energy resolved transverse magnetization densities $m_{1\mathbf{r}}(\epsilon), m_{2\mathbf{r}}(\epsilon)$ are obtained after integration over angles alone, and on the other hand the angle resolved magnetization tensors $m_{1\mathbf{r}}(\hat{\mathbf{p}}), m_{2\mathbf{r}}(\hat{\mathbf{p}})$ are obtained after energy integration. These quantities are defined through:

$$\begin{aligned} m_{1\mathbf{r}}(\mathbf{r}, t) &= -\mu_B \int d\tau g_{1\mathbf{r}}(\mathbf{r}, \mathbf{p}, t) \\ &= \frac{\tau_{sd}^2}{\ell_{sd}} \frac{e|E|}{m} \int d\epsilon m_{1\mathbf{r}}(\epsilon), \\ &= \frac{\tau_{sd}^2}{\ell_{sd}} \frac{e|E|}{m} \int \frac{d\hat{\mathbf{p}}}{4\pi} m_{1\mathbf{r}}(\hat{\mathbf{p}}), \end{aligned} \quad (29)$$

and similarly for $m_{2\mathbf{r}}(\mathbf{r}, t)$. Performing the angular integration, we obtain the following expression for the energy resolved transverse magnetization densities:

$$\begin{aligned} m_{1\mathbf{r}}(\epsilon) &= \frac{1+2\eta^2}{\eta} m_{\perp}(\epsilon) - \frac{1}{\eta} m_{\parallel}(\epsilon), \\ m_{2\mathbf{r}}(\epsilon) &= -m_{\perp}(\epsilon), \\ \text{with} & \\ m_{\perp}(\epsilon) &= -\mu_B \nu(\epsilon) \left(\frac{2}{3} \epsilon \partial_{\epsilon} f_{\perp} + f_{\perp} \right), \\ m_{\parallel}(\epsilon) &= -\mu_B \nu(\epsilon) f_{\parallel}^0. \end{aligned} \quad (30)$$

The form of $m_{\perp}(\epsilon)$ makes it clear why $\int d\epsilon m_{\perp}(\epsilon) = 0$. A possible very challenging experimental test of our approach would be to measure, with a magnetic STM tip, these effective itinerant electrons energy resolved transverse magnetic densities as a function of energy. In fact the existence of a finite $m_{2\mathbf{r}}(\epsilon)$ in the absence of any spin flip scattering would be a proof of the non trivial component f_{\perp} that is also at the origin of the equilibrium spin current.

Performing only the energy integration gives rise to the two angle resolved magnetic tensors $m_{2\mathbf{r}}(\hat{\mathbf{p}})$ and $m_{1\mathbf{r}}(\hat{\mathbf{p}})$:

$$\begin{aligned} m_{1\mathbf{r}}(\hat{\mathbf{p}}) &= \frac{1+3\eta^2}{\eta(1+\eta^2)} m_{\perp}(\delta^{ij} - 3\hat{\mathbf{p}}^i \hat{\mathbf{p}}^j) - \frac{1}{\eta} m_{\parallel} \delta^{ij}, \\ m_{2\mathbf{r}}(\hat{\mathbf{p}}) &= \frac{2\eta^2}{1+\eta^2} m_{\perp}(\delta^{ij} - 3\hat{\mathbf{p}}^i \hat{\mathbf{p}}^j), \\ \text{with} & \\ m_{\perp} &= -\mu_B \int d\epsilon \nu(\epsilon) f_{\perp}(\epsilon), \\ m_{\parallel} &= -\mu_B \int d\epsilon \nu(\epsilon) f_{\parallel}(\epsilon), \end{aligned} \quad (31)$$

These last two expressions show why a further angular integration cancels all the contributions induced by the existence of the non trivial component $f_{\perp}(\epsilon)$.

All these results are, in fact, valid for any spatial dimension provided we use the corresponding density of states and angular integration. They can also be generalized to an energy dependent scattering time with more complicated expressions for the distributions $g_{1,2\mathbf{r}}$. From the spin continuity equation, we know that we should find $m_{1\mathbf{r}} = -\frac{\tau_{sd}}{\ell_{sd}} j_{\parallel}$. This is indeed the case because we have the equality

$$\int d\tau g_{1i}^j(\mathbf{p}) = -\frac{\tau_{sd}}{\ell_{sd}} \int d\tau \frac{p^i}{m} g_{\parallel}^j(\mathbf{p}). \quad (32)$$

For an energy independent scattering time we furthermore obtain: $m_{1\mathbf{r}} = -\frac{\tau_{sd}}{\ell_{sd}} j_{\parallel} = -\frac{\tau_{sd}}{\ell_{sd}} \frac{e|E|\tau}{m} m_{\parallel}$.

Note that, although our results are non perturbative in η , they are valid order by order in η taking care of the fact that the leading order is $1/\eta$. This is in contrast with the linear response results of Kohno *et al.* where the standard "leading order term" leads to an unphysical magnetization component $m_{2\mathbf{r}}$ that is cancelled only when vertex corrections that constitute infinite order resummation are carefully taken into account.

We can partially generalize our results in two ways³⁴: (i) define distinct intraband scattering probabilities $w_{\mathbf{p},\mathbf{p}'}^{\pm}$ for energies ϵ_{\pm} . (ii) define anisotropic scattering probability $w_{\mathbf{p},\mathbf{p}'}$.

IV. SPIN-FLIP IMPURITY SCATTERING

A. Spin-basis invariant collision integral

To extend our Boltzmann approach to spin-flip scattering, a key step is to find a spin basis invariant formulation of a collision integral that characterizes a spin-flip process. We explain in details in the Appendix how to find this collision integral in the case of uniform magnet, and how to phenomenologically generalize it to non uniform magnets. The resulting collision integral is:

$$\hat{\mathcal{I}}^{\text{sf}} = \frac{1}{4} \int d\tau' \left[\hat{w}_{\mathbf{p},\mathbf{p}'}^{\text{sf}}(\mathbf{r}, t), [\hat{f}(\mathbf{p}', \mathbf{r}, t), 1 - \hat{f}(\mathbf{p}, \mathbf{r}, t)]_+ \right]_+ - \left[\hat{w}_{\mathbf{p},\mathbf{p}'}^{\text{sf}}(\mathbf{r}, t), [\hat{f}(\mathbf{p}, \mathbf{r}, t), 1 - \hat{f}(\mathbf{p}', \mathbf{r}, t)]_+ \right]_+,$$

with

$$\hat{w}_{\mathbf{p},\mathbf{p}'}^{\text{sf}}(\mathbf{r}, t) = \left[\frac{1}{2}(w_{\mathbf{p},\mathbf{p}'}^{\text{sf}+} + w_{\mathbf{p},\mathbf{p}'}^{\text{sf}-}) \hat{I} - \frac{1}{2}(w_{\mathbf{p},\mathbf{p}'}^{\text{sf}+} - w_{\mathbf{p},\mathbf{p}'}^{\text{sf}-}) \mathbf{n}(\mathbf{r}, t) \cdot \hat{\boldsymbol{\sigma}} \right],$$

and

$$\hat{f}(\mathbf{p}, \mathbf{r}, t) \equiv f(\mathbf{p}, \mathbf{r}, t) \hat{I} - \mathbf{f}_{\parallel}(\mathbf{p}, \mathbf{r}, t) \cdot \hat{\boldsymbol{\sigma}} \quad (33)$$

The fact that $\hat{f}(\mathbf{p}, \mathbf{r}, t)$ appears in the collision integral is quite natural since there is spin flip. On the other hand, the above form of $\hat{\mathcal{I}}^{\text{sf}}$ implies an effective vector spin flip probability $\hat{w}_{\mathbf{p},\mathbf{p}'}^{\text{sf}}$ that depends on the magnetization direction $\mathbf{n}(\mathbf{r}, t)$. Note that there is *a priori* no reason

to prevent contributions to $\hat{w}_{\mathbf{p},\mathbf{p}'}^{\text{sf}}(\mathbf{r}, t)$ that are linear in gradients of $\mathbf{n}(\mathbf{r}, t)$. But with our phenomenological approach there is no way to guess their specific form. This is in fact another key reason why a quantum approach using Keldysh Green function technique might be useful.

In the collision integral shown above, due to the appropriate antisymmetrization and for elastic scattering, all the terms that are quadratic in the distribution functions cancel. It then appears that in the equations of motion detailed in the Appendix, the effective spin flip collision integrals that appear for each component have the forms: $\mathcal{I}^{\text{sf}\pm} = \int d\tau' \frac{1}{2} (w_{\mathbf{p},\mathbf{p}'}^{\text{sf}+} \pm w_{\mathbf{p},\mathbf{p}'}^{\text{sf}-}) (f(\mathbf{p}') - f(\mathbf{p}))$ and $\mathcal{I}_{\alpha}^{\text{sf}\pm} = \int d\tau' \frac{1}{2} (w_{\mathbf{p},\mathbf{p}'}^{\text{sf}+} \pm w_{\mathbf{p},\mathbf{p}'}^{\text{sf}-}) (f_{\alpha}(\mathbf{p}') + f_{\alpha}(\mathbf{p}))$ for $\alpha = \parallel, 1, 2$.

In the following we shall consider only isotropic scattering in \mathbf{p} space ($w_{\mathbf{p},\mathbf{p}'}^{\text{sf}}(\epsilon) \equiv w^{\text{sf}}(\epsilon)$). Accordingly we define:

$$\frac{1}{\tau_{\pm}^{\text{sf}}(\epsilon_{\mathbf{p}})} = \int d\tau' w_{\mathbf{p},\mathbf{p}'}^{\text{sf}\mp} = w^{\text{sf}}(\epsilon_{\mathbf{p}}) \nu(\epsilon_{\mathbf{p}} \pm \hbar\omega_{sd}), \quad (34)$$

$$\eta_{\pm}^{\text{sf}}(\epsilon) = \frac{\tau_{sd}}{\tau_{\pm}^{\text{sf}}(\epsilon)},$$

We further define:

$$\frac{1}{\tau^{\text{sf}}} = \frac{1}{2} \left(\frac{1}{\tau_{+}^{\text{sf}}} + \frac{1}{\tau_{-}^{\text{sf}}} \right),$$

$$\frac{1}{\bar{\tau}} = \frac{1}{\tau} + \frac{1}{\tau^{\text{sf}}}, \quad (35)$$

$$p^{\text{sf}} = \frac{\tau_{+}^{\text{sf}} - \tau_{-}^{\text{sf}}}{\tau_{-}^{\text{sf}} + \tau_{+}^{\text{sf}}} < 1,$$

and correspondingly $\eta^{\text{sf}}, \bar{\eta}$ with $\eta^{\text{sf}} < \bar{\eta}$.

B. Zhang-Li relaxation time approximation

In order to recover equations of motion for the macroscopic quantities that correspond to the relaxation time approximation used by ZL, one needs three further assumptions:

(i)

$$\frac{1}{2} (w_{\mathbf{p},\mathbf{p}'}^{\text{sf}+} + w_{\mathbf{p},\mathbf{p}'}^{\text{sf}-}) \simeq \frac{1}{\tau^{\text{sf}}} \frac{1}{\nu(\epsilon_{\mathbf{p}})} \delta(\epsilon_{\mathbf{p}'} - \epsilon_{\mathbf{p}}),$$

and $\tau^{\text{sf}\pm}$ are constants independent of the energy $\epsilon_{\mathbf{p}}$;

(ii)

$$\frac{1}{2} (w_{\mathbf{p},\mathbf{p}'}^{\text{sf}+} - w_{\mathbf{p},\mathbf{p}'}^{\text{sf}-}) \simeq 0;$$

(iii) with assumption (i), at equilibrium, the collision integral of f_{\parallel} is no longer equal to zero at it should be. Therefore one needs to replace f_{\parallel} by $g_{\parallel} = f_{\parallel} - f_{\parallel}^0$ in the corresponding collision integral.

C. Extended relaxation time approximation

Because of assumption (ii) the usual relaxation time approximation ignores some important qualitative physics. In fact it is possible to relax (ii) because for the two components f, f_{\parallel} it is not necessary to make any approximation to obtain their exact forms in both equilibrium and out-of-equilibrium situations.

Indeed, by substituting the usual equilibrium form f^0, f_{\parallel}^0 , it is easily verified that both $\mathcal{I}^{\text{sf}\pm} \neq 0$ and $\mathcal{I}_{\parallel}^{\text{sf}\pm} \neq 0$. Nevertheless $\mathcal{I}^{\text{sf}+} + \mathcal{I}_{\parallel}^{\text{sf}-} = 0$ and $\mathcal{I}^{\text{sf}-} + \mathcal{I}_{\parallel}^{\text{sf}+} = 0$, thus the spin flip scattering does not modify the equilibrium properties of these two components.

By contrast, as we show in the Appendix in out-of-equilibrium situation, by relaxing (ii) the contributions g and g_{\parallel} are qualitatively modified. For the perpendicular components, without approximation (i) it is not possible to extract explicit closed forms for their distribution functions. Therefore in the following, as main assumption we assume that (i) is valid for the collision integral of the perpendicular components.

In the following we only give the final expressions for the distributions and physical quantities in both equilibrium and out of equilibrium situations. The detailed steps of the calculations are described in the Appendix.

D. equilibrium properties

At equilibrium f^0 and f_{\parallel}^0 are unmodified. For $f_{1\mathbf{r}}^0, f_{2\mathbf{r}}^0$ the previous equilibrium forms stay also valid but with a modified scattering time $\tau \rightarrow \bar{\tau}$. Thus at equilibrium the "p-isotropic" spin flip mechanism does not qualitatively change the physics of these two contributions.

Essential modifications arise for the last two components f_{1t}^0, f_{2t}^0 . At equilibrium they depend only on $\epsilon_{\mathbf{p}}$, therefore the energy and angular integrations of collision integrals can be performed and the two components are easily found :

$$\begin{aligned} f_{1t}^0(\epsilon) &= -\frac{1}{1 + (2\eta^{\text{sf}})^2} f_{\parallel}^0, \\ f_{2t}^0(\epsilon) &= -\frac{2\eta^{\text{sf}}}{1 + (2\eta^{\text{sf}})^2} f_{\parallel}^0 \end{aligned} \quad (36)$$

For energy independent η^{sf} the \mathbf{p} integration is immediate and we obtain two perpendicular components to the "equilibrium" magnetization: a component $m_{1t} = \frac{1}{1 + (2\eta^{\text{sf}})^2} m_{\parallel}$ collinear to $-\mathbf{n} \times \partial_t \mathbf{n}$ and a component $m_{2t} = 2\eta^{\text{sf}} m_{1t}$ collinear to $-\partial_t \mathbf{n}$. The modulus of this perpendicular magnetization vector is reduced by a factor $\frac{1}{\sqrt{1 + (2\eta^{\text{sf}})^2}}$ compared to the collisionless situation.

Once again, apart from this change of modulus, the spin flip has also induced a rotation in spin space of this transverse magnetization vector. Quantitatively, the ratio between the two transverse components of the magnetiza-

tion defines a new rotation angle $\tan \theta^{\text{sf}} = 2\eta^{\text{sf}} = \frac{2\tau_{sd}}{\tau^{\text{sf}}}$. We have previously pointed out that, as back action on the d electron local magnetization, the component m_{1t} contributes to the effective γ term¹¹. The new component $-\tau_{sd} m_{2t} \partial_t \mathbf{n}$ contributes to the effective Gilbert damping by a term

$$\begin{aligned} \alpha_{2t} &= \frac{\gamma \Delta_{sd} \tau_{sd}}{\mu_B M_s} \frac{2\eta^{\text{sf}}}{1 + (2\eta^{\text{sf}})^2} m_{\parallel} \\ &= \frac{2\tau_{sd} \tau^{\text{sf}}}{\tau^{\text{sf}2} + (2\tau_{sd})^2} \frac{\gamma \hbar P n_e}{M_s} \end{aligned} \quad (37)$$

This expression exactly coincides with the ZL result that was calculated using the macroscopic equations of motion in the relaxation time approximation. Taking only the leading order term in $\tau_{sd}/\tau^{\text{sf}}$ we also recover the expression of Kohno *et al* when the appropriate changes of notations are made.

E. out-of-equilibrium properties

To improve readability, the transport equations and the explicit calculation of the out equilibrium distributions of the different components are described in the Appendix. The resulting physical properties are the following.

For the particle and parallel spin currents, to first order in p^{sf} we obtain :

$$\begin{aligned} j &= \frac{n_e \bar{\tau} e \mathbf{E}}{m} (1 + p^{\text{sf}} \frac{\bar{\tau}}{\tau^{\text{sf}}} P), \\ J_{\parallel} &= \mu_B \frac{n_e \bar{\tau} e \mathbf{E}}{m} (P + p^{\text{sf}} \frac{\bar{\tau}}{\tau^{\text{sf}}}). \end{aligned} \quad (38)$$

Note that, even if there is no difficulty to obtain the results to any order in p^{sf} , the leading order term contains all the important qualitative modifications. These expressions show that the spin flip scattering modifies both the particle current and parallel current in a distinct manner. Such a result cannot be obtained using the standard relaxation time approximation and can only be introduced phenomenologically within the ZL macroscopic approach. We remark that, when $P = 1$ (fully polarized case), the parallel and particle current coincide as they should. When $P = 0$ (unpolarized) the parallel component is zero because p^{sf} is implicitly proportional to P (see the definition above). For later use, we also note that to first order in p^{sf} the relation between the parallel spin current and the charge current $j_e = e j$ is:

$$J_{\parallel} \simeq \frac{\mu_B}{e} (P + p^{\text{sf}} \frac{\bar{\tau}}{\tau^{\text{sf}}} (1 - P^2)) j_e \quad (39)$$

As explained in the Appendix, in out-of-equilibrium situation the spin current has also two transverse components collinear to $\partial_t \mathbf{n}$ and $\mathbf{n} \times \partial_t \mathbf{n}$, respectively. These two transverse components of the spin current are easily accessible with our formalism but cannot be explicitly

found with the ZL approach and need to be calculated within the linear response or Keldysh approach.

The last two components $g_{1,2\mathbf{r}}(\mathbf{p})$ of the out-of-equilibrium spin distribution function, as explained in the Appendix, can be recast in tensor form as $g_{1,2i}^j = \frac{\tau_{sd}^2}{\ell_{sd}} \frac{e|E|}{m} [\hat{p}^i \hat{p}^j x_{1,2}(\epsilon) + \delta^{ij} y_{1,2}(\epsilon)]$, with modified functions $x_{1,2}(\epsilon)$ and $y_{1,2}(\epsilon)$ as compared to the spin-conserving scattering case. With these expressions, by performing either angular integration or energy integration, we can calculate the modified energy resolved magnetic densities $m_{1\mathbf{r}}(\epsilon), m_{2\mathbf{r}}(\epsilon)$ (see Appendix) and angular resolved magnetic tensors $m_{1\mathbf{r}}(\hat{\mathbf{p}}), m_{2\mathbf{r}}(\hat{\mathbf{p}})$ respectively, and then by further integration the resulting physical magnetization components $m_{1\mathbf{r}}, m_{2\mathbf{r}}$. For comparison, we obtain the following expressions for the modified angular resolved magnetic tensors $m_{1\mathbf{r}}(\hat{\mathbf{p}}), m_{2\mathbf{r}}(\hat{\mathbf{p}})$:

$$\begin{aligned} m_{1\mathbf{r}}(\hat{\mathbf{p}}) &= \left(\frac{1+3\bar{\eta}^2}{\bar{\eta}(1+\bar{\eta}^2)} m_{\perp} + m_{\perp}^{\text{sf}} \right) (\delta^{ij} - 3\hat{\mathbf{p}}^i \hat{\mathbf{p}}^j) \\ &\quad - \frac{1}{\bar{\eta}} \frac{1}{1+(2\bar{\eta}^{\text{sf}})^2} \left(m_{\parallel} + p^{\text{sf}} \frac{\eta^{\text{sf}}}{\bar{\eta}} n_e \right) \delta^{ij}, \\ m_{2\mathbf{r}}(\hat{\mathbf{p}}) &= \left(\frac{1+2\bar{\eta}^2}{(1+\bar{\eta}^2)} m_{\perp} + \bar{\eta} m_{\perp}^{\text{sf}} \right) (\delta^{ij} - 3\hat{\mathbf{p}}^i \hat{\mathbf{p}}^j) \\ &\quad - \frac{1}{\bar{\eta}} \frac{2\eta^{\text{sf}}}{1+(2\bar{\eta}^{\text{sf}})^2} \left(m_{\parallel} + p^{\text{sf}} \frac{\eta^{\text{sf}}}{\bar{\eta}} n_e \right) \delta^{ij}, \end{aligned} \quad (40)$$

with

$$\begin{aligned} m_{\perp}^{\text{sf}} &= -\mu_B \int d\epsilon \nu(\epsilon) f_{\perp}^{\text{sf}}(\epsilon), \\ f_{\perp}^{\text{sf}}(\epsilon) &= p^{\text{sf}} \frac{\eta^{\text{sf}}}{\bar{\eta}} \frac{1}{1+\bar{\eta}^2} \frac{1}{\bar{\eta}} (f^0 - \frac{\hbar\omega_{sd}}{2} \partial_{\epsilon} f^0), \end{aligned}$$

Performing the angular integration we finally obtain the two transverse magnetization components:

$$\begin{aligned} m_{1\mathbf{r}} &= \frac{1}{1+(2\bar{\eta}^{\text{sf}})^2} \frac{\tau_{sd}}{\ell_{sd}} J_{\parallel}, \\ m_{2\mathbf{r}} &= \frac{2\eta^{\text{sf}}}{1+(2\bar{\eta}^{\text{sf}})^2} \frac{\tau_{sd}}{\ell_{sd}} J_{\parallel}. \end{aligned} \quad (41)$$

As back action on the localized d electron magnetization, the component $m_{1\mathbf{r}}$ gives rise to the first spin torque term and thus determines the parameter u in terms of the charge current density j_e :

$$\begin{aligned} u &= \frac{\gamma \Delta_{sd}}{\mu_B M_s} \ell_{sd} m_{1\mathbf{r}} \\ &= \frac{\tau^{\text{sf}^2}}{\tau^{\text{sf}^2} + (2\tau_{sd})^2} \left(1 + p^{\text{sf}} \frac{\bar{\tau}}{\tau^{\text{sf}}} \frac{1-P^2}{P} \right) \frac{\gamma \hbar P j_e}{e M_s}. \end{aligned} \quad (42)$$

The component $m_{2\mathbf{r}}$ gives rise to the second spin torque term and thus determines the parameter β as

$$\beta = 2 \frac{\tau_{sd}}{\tau^{\text{sf}}}. \quad (43)$$

This value of β coincides with ZL result (it therefore also coincides with the Kohno result for spin-isotropic spin flip when appropriate modifications of notations are made). Note that our result for the value of u cannot be explicitly calculated with the ZL macroscopic equations of motion. To leading order in $\frac{\tau_{sd}}{\tau^{\text{sf}}}$, the ratio $\beta/\alpha_{2t} \simeq M_s/m_{\parallel} \geq 1$ and thus, at first sight, it seems compatible with micro-magnetics and experiments. We note however that other

sources of dissipation (spin lattice relaxation for example) might give some contribution to the total effective Gilbert damping parameter α and thus modify the β/α ratio. It would therefore be interesting if experiments could provide measurements of the different contributions to the Gilbert damping parameter α .

V. SUMMARY, DISCUSSION AND PERSPECTIVES

Using the Landau-Sillin approach, we have studied the transport of electrons in the presence of an effective Zeeman field that has a space-time varying direction. The key ingredient is our Ansatz form of the spin density matrix that consists in a linear decomposition on quasistationary distribution functions along each possible direction provided by the first order space-time gradients of the magnetic field direction. We have shown step by step how the form of the different components of the distribution function is affected by the presence of spin-conserving and spin flip scattering, in both equilibrium and out-of-equilibrium situations. For spin-flip scattering we have defined a spin-basis-invariant collision integral and an extended relaxation time approximation that show the mixing of the particle and parallel components. Our calculations also illustrate the striking difference between a macroscopic quantity such as the transverse (perpendicular) magnetization component and its underlying distribution. This is particularly clear for the components $m_{2\mathbf{r}}$ and $f_{2\mathbf{r}}(\mathbf{p})$ in the presence of spin-conserving scattering in the out-of-equilibrium situation. The term $f_{2\mathbf{r}}(\mathbf{p})$ is non zero and leads to a finite energy resolved magnetization density $m_{2\mathbf{r}}(\epsilon)$ when only an angular integration $\hat{\mathbf{p}}$ is performed. But further energy integration of this density gives a zero contribution as expected from the spin conservation rule. More qualitatively and physically, we have clearly explained the existence of an equilibrium spin transverse current in the direction $\mathbf{n} \times \nabla_{\mathbf{r}} \mathbf{n}$ when there is no scattering at all. We have shown that, within the Boltzmann approach, the modulus of this equilibrium spin transverse current and its direction are affected by spin-conserving and spin flip scattering. The rotation is by the angle $\bar{\theta}$ and in the direction $\nabla_{\mathbf{r}} \mathbf{n}$, compared to the collisionless situation. Although we have not calculated it explicitly, we have shown that the out-of-equilibrium contribution to the transverse spin current is purely in the direction provided by the time derivative $\mathbf{n} \times \partial_t \mathbf{n}$ in the spin-conserving situation. In presence of spin flip this contribution is further rotated in a complicated manner towards the direction $\partial_t \mathbf{n}$. Concerning the transverse magnetization components, the situation is somewhat reversed. In "equilibrium" the transverse magnetization is along $\mathbf{n} \times \partial_t \mathbf{n}$ in both collisionless and spin-conserving scattering situations. In presence of spin flip it is rotated by an angle θ^{sf} to the direction $\partial_t \mathbf{n}$. For the localized d electron magnetization this rotation leads to the appearance of an effective Gilbert damping

correction term α_{2t} . In out of equilibrium situation, for spin-conserving scattering the transverse magnetization is along $\mathbf{n} \times \nabla_{\mathbf{r}}\mathbf{n}$ only and its modulus is directly proportional to the parallel spin current. In the presence of spin flip, this transverse magnetization is rotated by an angle θ^{sf} in the direction $\nabla_{\mathbf{r}}\mathbf{n}$. For the localized d electron magnetization these two components lead to the two spin transfer torque terms and thus allow determining the two parameters β and u . Our results for β and α_{2t} exactly with those of the ZL macroscopic approach and of Kohno *et al.* in the spin-isotropic spin flip scattering situation that we consider. Our approach allows to calculate explicitly the spin current polarization $P_j = u/u_e$ where u_e is the electron drift velocity that is purely phenomenological in ZL macroscopic approach.

We believe that this paper can be extended in at least four directions³⁵.

(i) For spin flip scattering, as we already pointed out, there is no reason to prevent terms proportional to the gradient of $\mathbf{n}(\mathbf{r}, t)$ in the spinor probability $\hat{w}_{\mathbf{p}, \mathbf{p}'}(\mathbf{r}, t)$. A natural extension of our work would consist in considering such terms by assuming some specific forms for the corresponding probability $\hat{w}_{\alpha, \mathbf{p}, \mathbf{p}'}$ ($\alpha = 1t, 1r, 2t, 2r$). In fact we believe that $\hat{w}_{1, 2r, \mathbf{p}, \mathbf{p}'}$ could be at the origin of an out-of-equilibrium spin Hall current.

(ii) In our paper we have only considered the most simple quadratic dispersion relation $\epsilon_{\mathbf{p}} = \frac{\mathbf{p}^2}{2m}$ and a \mathbf{p} independent effective Zeeman field. In the spirit of the work of J. Zhang *et al.*³⁶, it would be interesting to explore how far the equilibrium and out of equilibrium properties are changed for a general dispersion relation and a \mathbf{p} dependent effective Zeeman field.

(iii) It is possible to adapt our method to spin-valve systems³⁰. The key point consists in expanding the spinor distribution function $\hat{f}^\ell(\mathbf{p}, \mathbf{r}, t)$ in each region $\ell = L, R, C$ ($L \equiv$ left thick magnetic layer, $C \equiv$ central non magnetic layer and $R \equiv$ right thin magnetic layer) into the most general basis to first order in time gradients of the two magnetic layers directions $\mathbf{n}_{L, R}(t)$. If for simplicity one assumes that the "thick layer" direction \mathbf{n}_L is time independent, naively our Ansatz distribution for the thin layer would be:

$$\begin{aligned} \hat{f}^R(\mathbf{p}, \mathbf{r}, t) &= f^R(\mathbf{p}, \mathbf{r})\hat{I} + \mathbf{f}^R(\mathbf{p}, \mathbf{r}, t) \cdot \boldsymbol{\sigma}, \\ \text{with} \\ \mathbf{f}^R(\mathbf{p}, \mathbf{r}, t) &= f_{\parallel}^R(\mathbf{p}, \mathbf{r})\mathbf{n}_R(t) \\ &+ (f_{1r}^R(\mathbf{p}, \mathbf{r})\mathbf{n}_R \times \mathbf{n}_L + f_{2r}^R(\mathbf{p}, \mathbf{r})\mathbf{n}_L) \\ &+ \tau_{sd}^R (f_{1t}^R(\mathbf{p}, \mathbf{r})\mathbf{n}_R \times \partial_t \mathbf{n}_R + f_{2t}^R(\mathbf{p}, \mathbf{r})\partial_t \mathbf{n}_R) \\ &+ \tau_{sd}^R f_{3t}^R(\mathbf{p}, \mathbf{r})\mathbf{n}_L \times \partial_t \mathbf{n}_R \end{aligned} \quad (44)$$

In fact, preliminary results show that this Ansatz with an extended basis of six vectors is still insufficient³⁷.

(iv) In the context of ferromagnetic Fermi liquids, the Landau-Sillars approach has been used for a long time and equations of motion of the magnetization with terms similar to the spin torque terms have been established for example by Leggett²⁹. Nevertheless, there are still many questions that concern the transverse properties. To our

knowledge, most of the parametrizations used to study these systems²⁸ are similar to that of Kohno *et al.* and Tserkovnyak *et al.*. We thus believe that new insights can be provided by a parametrization similar to ours.

As a final remark and anticipating on our paper using Keldysh Green functions, we have pointed at several places that a collision integral is intrinsically a quantum object and that, therefore, quantum corrections might affect the results. The microscopic derivation of a collision integral requires the calculation of a self energy which itself depends on the Green function. The quantum object that plays a role similar to the spinor distribution $\hat{f}(\mathbf{r}, \mathbf{p}, t)$ is the time-space Wigner transform spinor Green function $\hat{G}(\mathbf{p}, \omega, \mathbf{r}, t)$. Very similarly to the spinor distribution $\hat{f}(\mathbf{r}, \mathbf{p}, t)$ we thus propose to adopt the following Ansatz form for the Green function:

$$\begin{aligned} \hat{G}(\mathbf{p}, \omega, \mathbf{r}, t) &= G(\mathbf{p}, \omega)\hat{I} + \mathbf{G}(\mathbf{p}, \omega, \mathbf{r}, t) \cdot \hat{\boldsymbol{\sigma}} \\ \text{with} \\ \mathbf{G}(\mathbf{p}, \omega, \mathbf{r}, t) &= G_{\parallel}(\mathbf{p}, \omega)\mathbf{n}(\mathbf{r}, t) \\ &+ \ell_{sd} [G_{1r}(\mathbf{p}, \omega)\mathbf{n} \times \nabla_{\mathbf{r}}\mathbf{n} + G_{2r}(\mathbf{p}, \omega)\nabla_{\mathbf{r}}\mathbf{n}] \\ &+ \tau_{sd} [G_{1t}(\mathbf{p}, \omega)\mathbf{n} \times \partial_t \mathbf{n} + G_{2t}(\mathbf{p}, \omega)\partial_t \mathbf{n}]. \end{aligned} \quad (45)$$

VI. APPENDIX

A. Spin-basis invariant collision integral for spin-flip scattering

Spin flip scattering in a uniform ferromagnet ($\mathbf{n}(\mathbf{r}, t) \equiv \mathbf{n}_0$) corresponds to an "interband" process. The collision integral of each eigen-spin distribution function $f_{\pm}(\mathbf{p}) = f(\mathbf{p}) \pm f_{\parallel}(\mathbf{p})$ is then:

$$\begin{aligned} \mathcal{I}_{\pm}^{\text{sf}} &= \int d\tau' w_{\mathbf{p}, \mathbf{p}'}^{\text{sf}\mp} [f_{\mp}(\epsilon_{\mathbf{p}'}) (1 - f_{\pm}(\epsilon_{\mathbf{p}})) \\ &\quad - f_{\pm}(\epsilon_{\mathbf{p}}) (1 - f_{\mp}(\epsilon_{\mathbf{p}'}))] \\ &= \int d\tau' w_{\mathbf{p}, \mathbf{p}'}^{\text{sf}\mp} (f_{\mp}(\epsilon_{\mathbf{p}'}) - f_{\pm}(\epsilon_{\mathbf{p}})), \end{aligned} \quad (46)$$

with $w_{\mathbf{p}, \mathbf{p}'}^{\text{sf}\mp} = w_{\mathbf{p}, \mathbf{p}'}^{\text{sf}}(\epsilon_{\mathbf{p}'}^{\mp})\delta(\epsilon_{\mathbf{p}}^{\mp} - \epsilon_{\mathbf{p}}^{\pm})$. The corresponding collision integrals for the particle density and parallel magnetization components are:

$$\begin{aligned} \mathcal{I}^{\text{sf}} &= \int d\tau' [\frac{1}{2}(w_{\mathbf{p}, \mathbf{p}'}^{\text{sf}+} + w_{\mathbf{p}, \mathbf{p}'}^{\text{sf}-})(f(\mathbf{p}') - f(\mathbf{p})) \\ &\quad + \frac{1}{2}(w_{\mathbf{p}, \mathbf{p}'}^{\text{sf}+} - w_{\mathbf{p}, \mathbf{p}'}^{\text{sf}-})(f_{\parallel}(\mathbf{p}') + f_{\parallel}(\mathbf{p}))], \\ \mathcal{I}_{\parallel}^{\text{sf}} &= \int d\tau' [-\frac{1}{2}(w_{\mathbf{p}, \mathbf{p}'}^{\text{sf}+} - w_{\mathbf{p}, \mathbf{p}'}^{\text{sf}-})(f(\mathbf{p}') - f(\mathbf{p})) \\ &\quad - \frac{1}{2}(w_{\mathbf{p}, \mathbf{p}'}^{\text{sf}+} + w_{\mathbf{p}, \mathbf{p}'}^{\text{sf}-})(f_{\parallel}(\mathbf{p}') + f_{\parallel}(\mathbf{p}))]. \end{aligned} \quad (47)$$

To extend the above results to a non uniform magnet we need first to find a spin-basis-invariant formulation such that for the uniform ferromagnet we can write a spin-matrix collision integral $\hat{\mathcal{I}} = \mathcal{I}\hat{I} + \mathbf{I} \cdot \hat{\boldsymbol{\sigma}}$ directly in terms of a spin-matrix distribution function $\hat{f}(\mathbf{p}) = f\hat{I} + \mathbf{f} \cdot \hat{\boldsymbol{\sigma}}$ (with $\mathbf{f} = f_{\parallel}\mathbf{n}_0$ for the uniform case). The following form of \mathcal{I} appears to be compatible with the above results for

\mathcal{I} and \mathcal{I}_{\parallel} :

$$\begin{aligned}\hat{\mathcal{I}}^{\text{sf}} &= \int d\tau' \hat{w}_{\mathbf{p},\mathbf{p}'}^{\text{sf}} [\hat{f}(\mathbf{p}') (1 - \hat{f}(\mathbf{p})) - \hat{f}(\mathbf{p}) (1 - \hat{f}(\mathbf{p}'))], \\ \text{with} \\ \hat{w}_{\mathbf{p},\mathbf{p}'}^{\text{sf}} &= [\tfrac{1}{2}(w_{\mathbf{p},\mathbf{p}'}^{\text{sf}+} + w_{\mathbf{p},\mathbf{p}'}^{\text{sf}-})\hat{I} - \tfrac{1}{2}(w_{\mathbf{p},\mathbf{p}'}^{\text{sf}+} - w_{\mathbf{p},\mathbf{p}'}^{\text{sf}-})\mathbf{n}_0 \cdot \hat{\boldsymbol{\sigma}}], \\ \text{and} \\ \hat{f}(\mathbf{p}) &\equiv f(\mathbf{p})\hat{I} - f_{\parallel}(\mathbf{p})\mathbf{n}_0 \cdot \hat{\boldsymbol{\sigma}}.\end{aligned}\quad (48)$$

On the one hand, the fact that $\hat{f}(\mathbf{p})$ appears in the collision integral is quite natural since there is spin flip, on the other hand the above form of $\hat{\mathcal{I}}^{\text{sf}}$ implies an effective vector spin flip probability $\hat{w}_{\mathbf{p},\mathbf{p}'}^{\text{sf}}$ that depends on the magnetization direction. We now phenomenologically extend the above form to space-time dependent magnetization with appropriate antisymmetrization to prevent imaginary terms:

$$\begin{aligned}\hat{\mathcal{I}}^{\text{sf}} &= \tfrac{1}{4} \int d\tau' \left[\hat{w}_{\mathbf{p},\mathbf{p}'}^{\text{sf}}(\mathbf{r}, t), [\hat{f}(\mathbf{p}', \mathbf{r}, t), 1 - \hat{f}(\mathbf{p}, \mathbf{r}, t)]_+ \right]_+ \\ &\quad - \left[\hat{w}_{\mathbf{p},\mathbf{p}'}^{\text{sf}}(\mathbf{r}, t), [\hat{f}(\mathbf{p}, \mathbf{r}, t), 1 - \hat{f}(\mathbf{p}', \mathbf{r}, t)]_+ \right]_+, \\ \text{with} \\ \hat{w}_{\mathbf{p},\mathbf{p}'}^{\text{sf}}(\mathbf{r}, t) &= [\tfrac{1}{2}(w_{\mathbf{p},\mathbf{p}'}^{\text{sf}+} + w_{\mathbf{p},\mathbf{p}'}^{\text{sf}-})\hat{I} \\ &\quad - \tfrac{1}{2}(w_{\mathbf{p},\mathbf{p}'}^{\text{sf}+} - w_{\mathbf{p},\mathbf{p}'}^{\text{sf}-})\mathbf{n}(\mathbf{r}, t) \cdot \hat{\boldsymbol{\sigma}}].\end{aligned}\quad (49)$$

B. Transport equation and out of equilibrium distribution functions in the presence of spin flip scattering

Using the form of the collision integrals introduced in the previous section, the equations of motion of all components in the presence of both spin-flip and spin-conserving impurities now read:

$$\begin{aligned}e\mathbf{E}\nabla_{\mathbf{p}}f &= \mathcal{I} + \mathcal{I}^{\text{sf}+} + \mathcal{I}_{\parallel}^{\text{sf}-}, \\ e\mathbf{E}\nabla_{\mathbf{p}}f_{\parallel} &= \mathcal{I}_{\parallel} - \mathcal{I}^{\text{sf}-} - \mathcal{I}_{\parallel}^{\text{sf}+}, \\ e\mathbf{E}\nabla_{\mathbf{p}}f_{2\mathbf{r}} + \tfrac{1}{\ell_{sd}}(\tfrac{\mathbf{p}}{m}f_{\parallel} - \tfrac{\hbar\omega_{sd}}{2}\nabla_{\mathbf{p}}f) + \omega_{sd}f_{1\mathbf{r}} &= \mathcal{I}_{2\mathbf{r}} - \mathcal{I}_{2\mathbf{r}}^{\text{sf}+}, \\ e\mathbf{E}\nabla_{\mathbf{p}}f_{1\mathbf{r}} - \omega_{sd}f_{2\mathbf{r}} &= \mathcal{I}_{1\mathbf{r}} - \mathcal{I}_{1\mathbf{r}}^{\text{sf}+}, \\ e\mathbf{E}\nabla_{\mathbf{p}}f_{2t} + \tfrac{1}{\tau_{sd}}f_{\parallel} + \omega_{sd}f_{\perp t1} &= \mathcal{I}_{2t} - \mathcal{I}_{2t}^{\text{sf}+}, \\ e\mathbf{E}\nabla_{\mathbf{p}}f_{1t} - \omega_{sd}f_{2t} &= \mathcal{I}_{1t} - \mathcal{I}_{1t}^{\text{sf}+},\end{aligned}\quad (50)$$

with: $\mathcal{I}^{\text{sf}\pm} = \int d\tau' \tfrac{1}{2}(w_{\mathbf{p},\mathbf{p}'}^{\text{sf}+} \pm w_{\mathbf{p},\mathbf{p}'}^{\text{sf}-})(f(\mathbf{p}') - f(\mathbf{p}))$ and $\mathcal{I}_{\alpha}^{\text{sf}\pm} = \int d\tau' \tfrac{1}{2}(w_{\mathbf{p},\mathbf{p}'}^{\text{sf}+} \pm w_{\mathbf{p},\mathbf{p}'}^{\text{sf}-})(f_{\alpha}(\mathbf{p}') + f_{\alpha}(\mathbf{p}))$ for $\alpha = \parallel, 1, 2$.

The equilibrium properties are easily found and were described in the main text. For the out of equilibrium properties, within our extended relaxation time approx-

imation the equations of motion become:

$$\begin{aligned}e\mathbf{E}\nabla_{\mathbf{p}}f^0 &= -\tfrac{1}{\bar{\tau}}(g - p^{\text{sf}}\tfrac{\bar{\tau}}{\tau_{\text{sf}}}g_{\parallel}) + \tfrac{1}{\bar{\tau}}\int \tfrac{d\mathbf{p}'}{4\pi}(g(\mathbf{p}') + p^{\text{sf}}\tfrac{\bar{\tau}}{\tau_{\text{sf}}}g_{\parallel}(\mathbf{p}')) \\ e\mathbf{E}\nabla_{\mathbf{p}}f_{\parallel}^0 &= -\tfrac{1}{\bar{\tau}}(g_{\parallel} - p^{\text{sf}}\tfrac{\bar{\tau}}{\tau_{\text{sf}}}g) \\ &\quad + \tfrac{1}{\bar{\tau}}\int \tfrac{d\mathbf{p}'}{4\pi}(1 - 2\tfrac{\bar{\tau}}{\tau_{\text{sf}}})g(\mathbf{p}') - p^{\text{sf}}\tfrac{\bar{\tau}}{\tau_{\text{sf}}}g(\mathbf{p}')) \\ e\mathbf{E}\nabla_{\mathbf{p}}f_{2t}^0 + \tfrac{1}{\tau_{sd}}g_{\parallel} + \omega_{sd}g_{\perp t1} &= -\tfrac{1}{\bar{\tau}}g_{2t} \\ &\quad + \tfrac{1}{\bar{\tau}}\int \tfrac{d\mathbf{p}'}{4\pi}(1 - 2\tfrac{\bar{\tau}}{\tau_{\text{sf}}})g_{2t}(\mathbf{p}') \\ e\mathbf{E}\nabla_{\mathbf{p}}f_{1t}^0 - \omega_{sd}g_{2t} &= -\tfrac{1}{\bar{\tau}}g_{1t} + \tfrac{1}{\bar{\tau}}\int \tfrac{d\mathbf{p}'}{4\pi}(1 - 2\tfrac{\bar{\tau}}{\tau_{\text{sf}}})g_{1t}(\mathbf{p}') \\ e\mathbf{E}\nabla_{\mathbf{p}}f_{2\mathbf{r}}^0 + \tfrac{1}{\ell_{sd}}(\tfrac{\mathbf{p}}{m}g_{\parallel} - \tfrac{\hbar\omega_{sd}}{2}\nabla_{\mathbf{p}}g) + \omega_{sd}g_{1\mathbf{r}} &= -\tfrac{1}{\bar{\tau}}g_{2\mathbf{r}} \\ &\quad + \tfrac{1}{\bar{\tau}}\int \tfrac{d\mathbf{p}'}{4\pi}(1 - 2\tfrac{\bar{\tau}}{\tau_{\text{sf}}})g_{2\mathbf{r}}(\mathbf{p}') \\ e\mathbf{E}\nabla_{\mathbf{p}}f_{1\mathbf{r}}^0 - \omega_{sd}g_{2\mathbf{r}} &= -\tfrac{1}{\bar{\tau}}g_{1\mathbf{r}} + \tfrac{1}{\bar{\tau}}\int \tfrac{d\mathbf{p}'}{4\pi}(1 - 2\tfrac{\bar{\tau}}{\tau_{\text{sf}}})g_{1\mathbf{r}}(\mathbf{p}')\end{aligned}\quad (51)$$

The first two equations show that in this extended relaxation time approximation g and g_{\parallel} are both linear combinations of $\nabla_{\mathbf{p}}f^0$ and $\nabla_{\mathbf{p}}f_{\parallel}^0$. As a consequence in the first four equations the terms with angular integration over $\hat{\mathbf{p}}'$ do not contribute. To solve these equations it is convenient to further split each g_{α} into $g_{\alpha} = \bar{g}_{\alpha} + g_{\alpha}^{\text{sf}}$ where \bar{g}_{α} is obtained, from the expressions found in the spin conserving case, by the substitution $\tau \rightarrow \bar{\tau}$.

To simplify the expressions and since it does not qualitatively modify the results, we only retain the first order contribution in p^{sf} although there is no difficulty to find the exact forms. For the first four components, we obtain:

$$\begin{aligned}g^{\text{sf}}(\mathbf{p}) &= -p^{\text{sf}}\tfrac{\eta^{\text{sf}}}{\bar{\eta}}\bar{\tau}e\mathbf{E}\nabla_{\mathbf{p}}f_{\parallel}^0, \\ g_{\parallel}^{\text{sf}}(\mathbf{p}) &= -p^{\text{sf}}\tfrac{\eta^{\text{sf}}}{\bar{\eta}}\bar{\tau}e\mathbf{E}\nabla_{\mathbf{p}}f^0, \\ g_{1t}^{\text{sf}}(\mathbf{p}) &= \tfrac{1}{1+\bar{\eta}^2}\bar{\tau}e\mathbf{E}\left[p^{\text{sf}}\tfrac{\eta^{\text{sf}}}{\bar{\eta}}\nabla_{\mathbf{p}}f^0 + \tfrac{2\eta^{\text{sf}}}{1+(2\eta^{\text{sf}})^2}(1-\bar{\eta})\bar{\eta}\nabla_{\mathbf{p}}f_{\parallel}^0\right], \\ g_{2t}^{\text{sf}}(\mathbf{p}) &= \tfrac{\bar{\eta}}{1+\bar{\eta}^2}\bar{\tau}e\mathbf{E}\left[p^{\text{sf}}\tfrac{\eta^{\text{sf}}}{\bar{\eta}}\nabla_{\mathbf{p}}f^0 + \tfrac{2\eta^{\text{sf}}}{1+(2\eta^{\text{sf}})^2}(1+\bar{\eta})\bar{\eta}\nabla_{\mathbf{p}}f_{\parallel}^0\right]\end{aligned}\quad (52)$$

Each of these components gives rise to some finite current only (particle or spin current).

For the last two components $g_{1,2\mathbf{r}}$ we remind that the terms with angular integration over $\hat{\mathbf{p}}'$ contribute. Once again we write $g_{1,2i}^j = \tfrac{\tau_{sd}^2}{\ell_{sd}}\tfrac{e|E|}{m}[\hat{p}^i\hat{p}^j x_{1,2}(\epsilon) + \delta^{ij}y_{1,2}(\epsilon)]$ and split accordingly $x_{1,2}$, $y_{1,2}$ into $x_{1,2} = \bar{x}_{1,2} + x_{1,2}^{\text{sf}}$ and

$y_{1,2} = \bar{y}_{1,2} + y_{1,2}^{\text{sf}}$. We then obtain:

$$\begin{aligned} x_1^{\text{sf}} &= 2\epsilon\partial_\epsilon f_\perp^{\text{sf}}, \\ x_2^{\text{sf}} &= \bar{\eta}x_1^{\text{sf}}, \\ y_1^{\text{sf}} &= -\frac{x_1^{\text{sf}}}{3} - \frac{1}{1+(2\eta^{\text{sf}})^2} \left[2\eta^{\text{sf}}(\bar{y}_2 + \frac{\bar{x}_2}{3}) + (2\eta^{\text{sf}})^2(\bar{y}_1 + \frac{\bar{x}_1}{3}) \right. \\ &\quad \left. - (1+\bar{\eta}^2)(\frac{x_\perp^{\text{sf}}}{3} + f_\perp^{\text{sf}}) + p^{\text{sf}}\frac{\eta^{\text{sf}}}{\bar{\eta}}\frac{1}{\bar{\eta}}f^0 \right], \\ y_2^{\text{sf}} &= -\bar{\eta}\frac{x_1^{\text{sf}}}{3} - \frac{2\eta^{\text{sf}}}{1+(2\eta^{\text{sf}})^2} \left[2\eta^{\text{sf}}(\bar{y}_2 + \frac{\bar{x}_2}{3}) \right. \\ &\quad \left. - (\bar{y}_1 + \frac{\bar{x}_1}{3}) - (1+\bar{\eta}^2)(\frac{x_\perp^{\text{sf}}}{3} + f_\perp^{\text{sf}}) + p^{\text{sf}}\frac{\eta^{\text{sf}}}{\bar{\eta}}\frac{1}{\bar{\eta}}f^0 \right], \end{aligned}$$

with

$$f_\perp^{\text{sf}} = p^{\text{sf}}\frac{\eta^{\text{sf}}}{\bar{\eta}}\frac{1}{1+\bar{\eta}^2}\frac{1}{\bar{\eta}}(f^0 - \frac{\hbar\omega_{sd}}{2}\partial_\epsilon f_\parallel^0), \quad (53)$$

With these expressions, by performing the angular integration we obtain the following modified expressions for the energy resolved transverses magnetization densities $m_{1\mathbf{r}}(\epsilon), m_{2\mathbf{r}}(\epsilon)$:

$$\begin{aligned} m_{1\mathbf{r}}(\epsilon) &= \frac{1}{1+(2\eta^{\text{sf}})^2} \left(\frac{1+2\bar{\eta}^2+2\eta^{\text{sf}}\bar{\eta}}{\bar{\eta}}m_\perp(\epsilon) + (1+\bar{\eta}^2)m_\perp^{\text{sf}}(\epsilon) \right. \\ &\quad \left. - \frac{1}{\bar{\eta}}(m_\parallel(\epsilon) + p^{\text{sf}}\frac{\eta^{\text{sf}}}{\bar{\eta}^2}n_e(\epsilon)) \right), \\ m_{2\mathbf{r}}(\epsilon) &= -m_\perp + 2\eta^{\text{sf}}m_{1\mathbf{r}}(\epsilon), \\ \text{with} \\ m_\perp^{\text{sf}}(\epsilon) &= \nu(\epsilon)(\frac{2}{3}\epsilon\partial_\epsilon f_\perp^{\text{sf}} + f_\perp^{\text{sf}}), \\ n_e(\epsilon) &= \nu(\epsilon)f^0(\epsilon), \end{aligned} \quad (54)$$

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 - ³³ Mathematically, the extended basis constitutes a linearly independent basis in the space of adiabatic functions $a(\mathbf{p}, \mathbf{r}, t)$ defined by $a(\mathbf{p}, \mathbf{r}, t) = a_1\mathbf{n}(\mathbf{r}, t) + a_2\mathbf{n} \times \nabla_{\mathbf{r}}\mathbf{n} + a_3\nabla_{\mathbf{r}}\mathbf{n} + a_4\mathbf{n} \times \partial_t\mathbf{n} + a_5\partial_t\mathbf{n}$, where the $a_\alpha(\mathbf{p})$ are function of \mathbf{p} only; i.e $a(\mathbf{p}, \mathbf{r}, t) = 0$ implies $a_\alpha(\mathbf{p}) = 0$ for all α and $a(\mathbf{p}, \mathbf{r}, t) = b(\mathbf{p}, \mathbf{r}, t)$ implies $a_\alpha(\mathbf{p}) = b_\alpha(\mathbf{p})$.
 - ³⁴ For spin conserving scattering we can generalize our work in two ways: (i) define distinct intraband scattering probabilities $w_{\mathbf{p},\mathbf{p}'}^\pm$ for energies ϵ_\pm . (ii) define anisotropic scattering probabilities $w_{\mathbf{p},\mathbf{p}'}$.
(i) In establishing the parametric forms $f_{1,2}$ we do not need the explicit forms of f^0 and $f_\parallel^0(\epsilon_{\mathbf{p}})$. We only need that $f^0(\epsilon_{\mathbf{p}})$ and $f_\parallel^0(\epsilon_{\mathbf{p}})$ depend only on $\epsilon_{\mathbf{p}}$. But in fact quite generally we know that $f^0(\epsilon_{\mathbf{p}}) = \frac{1}{2}(A_+(\epsilon_{\mathbf{p}}^+) + A_-(\epsilon_{\mathbf{p}}^-))$ and $f_\parallel(\epsilon_{\mathbf{p}}) = \frac{1}{2}(A_{\parallel+}(\epsilon_{\mathbf{p}}^+) + A_{\parallel-}(\epsilon_{\mathbf{p}}^-))$. Clearly the components $f_{1,2}(\epsilon_{\mathbf{p}})$ can also be split in two components $A_{1,2\pm}(\epsilon_{\mathbf{p}}^\pm)$. Using this property, an immediate generalization of the above results consists in separating the collision integral into $\mathcal{I}_\alpha = \mathcal{I}_\alpha^{++} + \mathcal{I}_\alpha^{--}$ (with obvious notations) and to

specify distinct probabilities $w_{\mathbf{p},\mathbf{p}'}^\pm = w_{\mathbf{p},\mathbf{p}',\epsilon_\pm}^\pm \delta(\epsilon_\pm^\pm - \epsilon_{\mathbf{p}'}^\pm)$. In that way, for isotropic scattering, all the previous expressions are valid when interpreted for each index \pm , and this allows considering the possibility of two distinct scattering times τ_\pm .

(ii) Anisotropic scattering consists in writing $w_{\mathbf{p},\mathbf{p}'} = \sum_\ell w_{\epsilon_\mathbf{p}}^\ell P_\ell(\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}')$ where $P_\ell(\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}')$ are Legendre polynomials of order ℓ . In the equilibrium situation, the only modification of our results consists in replacing the isotropic scattering time $1/\tau = w^0(\epsilon)\nu(\epsilon)$ by the transport scattering time $1/\tau_{\text{tr}} = (w^0(\epsilon) - \frac{1}{3}w^1(\epsilon))\nu(\epsilon)$. In the out-of-equilibrium situation the analysis becomes more subtle and

is beyond the scope of the present work.

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³⁷ It appears in fact that, to describe a time dependent effect to linear order, it is necessary to consider an extended basis of the nine following vectors $\mathbf{n}_L, \mathbf{n}_R, \mathbf{n}_L \times \mathbf{n}_R$ and the six new vectors $\partial_t \mathbf{n}_R, \mathbf{n}_L \times \partial_t \mathbf{n}_R, \mathbf{n}_R \times \partial_t \mathbf{n}_R, \mathbf{n}_L \times (\mathbf{n}_L \times \partial_t \mathbf{n}_R), \mathbf{n}_L \times (\mathbf{n}_R \times \partial_t \mathbf{n}_R)$ and $\mathbf{n}_L \times (\mathbf{n}_L \times (\mathbf{n}_R \times \partial_t \mathbf{n}_R))$. In this basis the mathematical properties described in³³ are valid.